Mineral Rights Investment Risk Management Model and Implementation Based on Empirical Data

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Abstract-This paper investigates the risk and uncertainties of the mineral reserves and future prices in mining rights investment. Based on random variable of reserves and prices, a practical stochastic method is introduced to fit the distribution functions of standardized parameters. The proposed methods are then verified by international gold prices and demonstrated as a case study. The past forecast and probability distribution show high level of consistency with 2005 to present gold price trend based on empirical data between 1968 and 2000. The generalization of the model make it relatively simple to be adopted by other type of mineral investment with minor modification.

Keywords: *Exploration Rights, Resource Reserves, Forecast Prices, Probability Distribution*

1. Introduction

Since the establishment of Chinese mining rights market in the 1990s, mining rights have been gradually transformed from state-own assets into an essential industry of Chinese market economy. Later in the transformation, mineral resources start to enter the secondary market. The transactions of the resources are no longer exclusively involving direct investment in actual mineral, but in the form of shares of mineral rights, similar to stock market. However, mining rights trade features its own unique characteristics and high level of complexity. Compare to other risk management, it presents higher technical requirement thanks to the uncertainty of mining and higher capital requirement due to the scale of mining industry. Both the risk and benefit in mining investments exceed other financial products.

The main rationale behind the uncertainties is a complex combination and interactions of resource reserves, market demand and the future price projection. Without proper quantification, it is infeasible to correctly understand such correlations. This work focuses on these uncertainties and provides a practical methods to measure the uncertainty of reserves and prices. These methods may help investors to recognize the risk and adjust their investment behaviors appropriately. It is also the fundamental step to establish risk assessment model in mining investment. This research is significant for mining rights market decision making with a wide range of applications.

Similar to the rest of the world, the Chinese mining rights is divided into exploration rights and mineral rights. Firstly, the government mining agency provides access to the exploration rights to investors. After two years of pilot phase, the investor is allowed to trade the rights in secondary market. The prospecting investor faces two major types of uncertainty, one is "underground risk" caused by inadequate understanding of reserves situation, which will become more accurate with in-depth exploration. The other type is the "above-the-ground risk" which is due to the fluctuation of future price in mining resources market. To investigate the overall risks, the analysis of both types are important. In this work, analysis on both risks are conducted using simple statistical method. With the random variables distribution of reserves and prices, the risk of exploration rights investment could be assessed properly. This work is presented in the following sections. In section 2 of the paper, the parameters of the model are analyzed and standardized. Section 3 presents the model creation and the key criteria of the risk evaluation process. The results based on a case study of gold trade between 1968 and 2000 are illustrated in section 4. The conclusions and targeted future works are presented in section 5.

2. Parameter Standardization

In initial phase of exploration, investors usually focus on the reserves, due to the fact if the amount and quality (degree) of reserves can not be estimated with certain accuracy, the results of the investment will be catastrophic. How to describe the probability distribution of reserves is a complex problem, the data acquisition will become another obstacle. Based on past experience, choosing the correct type of data has become the most critical aspect in this issue.

The previous works in this area include R.J. Gilbert's work, which assumes the reserves following certain subjective probability distributions[1], simple Bayes method is utilized to calculate the conditional probability. K.J. Arrow and S. Chang, S.D. Deshmukh and S.R. Pliska assume reserves follow a random distribution, in unit area it contains reserves obtain Poisson distribution with known parameters [2],[3]. R.S. Pindyck investigated the the optimal exploration model, analyzed the Hotelling rule under random conditions. However, the method lacks considerations on interactions between the two types of risks [4]. N.V. Quyen introduced the concept of correlations between the exploration and exploitation [5]. D. Martin and F.T. Sparrow noticed the two operations are closely correlated, and proposed the process to create the "simulation-optimization" model[6].

Reserves of mineral resources is always an absolute value, however, the assessment results of the reserves are uncertain, which change along with the development of exploration process, the geological knowledge and the deposit recover cost index of the targeted mining area. It also depends on the use of different evaluation methods and software. For the investor, it is important to obtain the maximum amount of information on mineral reserves with minimum exploration efforts before further exploration or transaction occur. Meanwhile, for the exploitation rights buyer, the attentions are on the credibility of main reserves the seller of exploration rights provides.

Currently, structure curve integral and dynamic fractal method (SD method) is an advanced estimation and validation method for mineral reserves. SD could estimate reserves in different exploration stage and precision expectations. Although SD precision shows the accuracy of reserves, it defines the size of reserves with an error range for the estimated reserves number. With more available data, the accuracy of the estimation increases, this leads to the decimate of SD precision range. Vise versa, the less the data, the wider range of SD precision target. For example: the SD accuracy of 0.4 means that the estimation precision and engineering feasibility index reach 40%, the maximum reserves error is up to 60%. Different from general $\pm$ 60% error, the multiplication and division are used to calculate the error. Assuming the exploration reserve is 100 million tons, the error is 60% (i.e. a accuracy of 0.4), the lower limit of precision range is 1 million tons $(1 - 60\%) = 0.4$ million tons, and the upper limit of precision range is 1 million tons $\div\ 40\% = 2.5$ million tons. In another words, the precision is between 0.4 times and the $(1 \div 0.4)$ times of the estimation.

Once such mineral resources are purchased(SD accuracy = 0.4 and 1 million tons estimation), it is obvious that investors would require more knowledge about the probability distribution between 0.4 and 2.5 million tons. What is the probability for reserves of 0.8 million tons, 0.6∽0.7 million tons, or higher than 2 million tons? Does a description of the probability distribution exist? The rest of the section provides the specific parameters to determine the distribution function of mineral reserves.

2.1 Data Selection

To characterize the distribution function, selected empirical data with specific investigation value are introduced. Assume n deposits of same minerals (e.g. copper) exist in the mining rights covered area at the beginning of the exploration. This gives estimated reserves q_{ij}, $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, 100)$ and error range $[q_{ij} \cdot t_j \ , \ q_{ij} \cdot t_j^{-1}]$, $(j = 1, 2, \ldots, 100)$ where t_j is the SD accuracy. 10 precision values from 10% to 100% are selected in the simulation. Accuracy 100% is considered to be hypothetical confirmed reserves. The possibilities are shown in an array in table 1.

Table 1: Estimated Reserves with Various Accuracy

Accuracy	10%	20%	....	90%	100% (Confirmed Reserve)
Mineral 1	q_{11}	q_{12}	$\cdots$	q_{19}	q_{10}^*
Mineral 2	q_{21}	q_{22}	$\cdots$	q_{29}	q_{20}^*
...	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Mineral n	q_{n1}	q_{n2}	$\cdots$	q_{n9}	q_{n0}^*

The different scale of mineral deposits means the error may be considerably large, so the estimated data in table 1 can not be used directly, we calculate the relative bias between estimated reserves under different stages until reaching 100% proven reserves ${q_{i0}}^*$. The sample data are normalized as in the following equation.

$$q_{Bias} = \frac{q_{ij} - {q_{i0}}^*}{{q_{i0}}^*}, (i = 1, 2, \cdots, n) \tag{1}$$

The results of the normalization are shown in table 2.

Table 2: Relative Bias based Estimated Reserves

Accuracy	10%	20%	....	90%	100% (Confirmed Reserve)
Mineral 1	$\frac{(q_{11}-q_{10}^*)}{q_{10}^*}$	$\frac{(q_{12}-q_{10}^*)}{q_{10}^*}$	$\cdots$	$\frac{(q_{19}-q_{10}^*)}{q_{10}^*}$	q_{10}^*
Mineral 2	$\frac{(q_{21}-q_{20}^*)}{q_{20}^*}$	$\frac{(q_{22}-q_{20}^*)}{q_{20}^*}$	$\cdots$	$\frac{(q_{29}-q_{20}^*)}{q_{20}^*}$	q_{20}^*
...	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Mineral n	$\frac{(q_{n1}-q_{n0}^*)}{q_{n0}^*}$	$\frac{(q_{n2}-q_{n0}^*)}{q_{n0}^*}$	$\cdots$	$\frac{(q_{n9}-q_{n0}^*)}{q_{n0}^*}$	q_{n0}^*

The relative bias data are considered as distribution fitting sample data. In the next subsection, distribution of relative bias for estimated reserves will be characterized.

2.2 Distribution fitting

Given the accuracy $t_j\%$, the relative bias of reserves is used as the test data. Firstly, sample data are sorted in ascending order, divide $(-\infty, +\infty)$ into m intervals which are mutually exclusive, i.e. choose $a_i, (i = 1, 2, \ldots, m-1)$, which $(-\infty < a_1 < \cdots < a_i < a_{i+1} < +\infty)$. Also note that, $A_1 = (-\infty, A_1), \ldots, A_i = [a_{i-1}), i = 2, \ldots, m-2, A_m = [a_{m-1}, +\infty)$. By doing so, the frequency N_i and its corresponding value $n_i, i = 1, 2, \ldots, m$ can be calculated. Based on these results, the histogram is then drown. The probability distributed density function is given as $f_j(\frac{q_{ij}-q_{i0}{}^*}{q_{i0}{}^*}|t_j\%), (j = 1, 2, \cdots, n)$. Once the probability distributed density function is obtained, by using estimated reserves q_{ij} (obtained by SD) as mean value under certain accuracy $t_j\%$, we measured the distribution of forecast reserves in the relative bias interval $[\frac{t_j}{100} \cdot q_{ij}, \frac{q_{ij}}{t_j\%}]$. The characterization of the distribution function of estimated reserves is $F(q|t_j\%) = \int_{t_j\%\cdot q_{ij}}^{q} f_i(u|t_j\%)du$. In this way, the reserves estimation probability of reserves could be calculated in any probability.

3. Stochastic Characterization of Mineral Prices

The mineral price variations and projection is a research area draws many interests. R.S. Pindyck investigated the optimal production problem based on conditions assume random variation and mineral prices are determined by external factors [7]. M. Carlson, Z. Khokher and S. Titman consider the pricing and exploration strategy as self determining variables, and created a resource-market balance model [8]. In this paper, we consider the market price and exploration strategy are both determining variables as a new concept for model creation. As prospecting investors, the future price that is another major problems along with reserves information. The future minerals price can be predicted by its time series data, but the price affected by market demand and policy factor may fluctuate. The investors often find the statistical pattern of future price trend. The next research is how to illustrate the probability distribution of the future minerals price. We use reverse thinking of time series forecasting to overcome data inadequacy.

In order to characterize the volatility of minerals transaction price, we select a certain mineral right price, such as $p_1, p_2, \ldots, p_{t-2}, p_{t-1}, p_t$ use time series analysis method (like ARMA and ARCH forecasting model) to predict the expectation $\hat{P}_{n+k}$ and variance σ^2 of the future price. The predicted value by time series analysis can be calculated by inertial extrapolation in light of historical data changing regularity, but the true price affected by policy shift and market demand changes will bring about deviation with predicted price, whose level imply the domain of external variable influence. Therefore, we choose the deviation data series of the past predicted value and true value as the sample data to find the impact pattern of future price fluctuation and provide the distribution function of random variable $\hat{P}_{n+k}$. Specific measures are introduced below:

(1) Select time series data of a certain minerals historical transaction price $p_1, p_2, \cdots, p_{t-2}, p_{t-1}, p_t$;
(2) Use time series analysis to forecast the future price $\hat{P}_{t+1}$;
(3) Calculate p_t and use time series data $p_1, p_2, \cdots, p_{t-2}, p_{t-1}$ to forecast $\hat{P}_t$;
(4) Calculate p_{t-1} and use time series data $p_1, p_2, \cdots, p_{t-2}$ to forecast $\hat{P}_{t-1}$;
(5) Repeat the step(3) m times, we obtain:$\hat{p}_{t-m}, \hat{p}_{t-m-1}, \cdots, \hat{p}_{t-2}, \hat{p}_{t-1}, \hat{p}_t$;
(6) Calculate the deviation between predicted data and real data, $\hat{p}_{t-m}-p_{t-m}, \hat{p}_{t-m-1}-p_{t-m-1}, \cdots, \hat{p}_{t-2}-p_{t-2}, \hat{p}_{t-1}-p_{t-1}, \hat{p}_t-p_t$;

As the sample data for determining the distribution function of the future predicted price been selected, these deviation data include the information that price fluctuations is influenced by external variables.The fitting of the distribution is achieved in the following 3 steps:

(1) Rank the above selected sample data $\hat{p}_{t-m}-p_{t-m}, \hat{p}_{t-m-1}-p_{t-m-1}, \cdots, \hat{p}_{t-2}-p_{t-2}, \hat{p}_{t-1}-p_{t-1}, \hat{p}_t-p_t$ in ascending order. Dividing $(-\infty, +\infty)$ into m intervals, that is, select b_i, for $i = 1, \cdots, m-1$, so as to $(-\infty < b_1 < \cdots < b_i < b_{i+1} < \cdots < b_{m-1} < +\infty)$. Denote $B_1 = (-\infty, b_1), B_i = [b_{i-1}, b_i), i = 2, \ldots, m-2, B_m = [b_{m-1}, +\infty)$, calculate the frequency for each of interval $n_i, i = 1, 2, \cdots, m, \sum^m n_i = m+1$, then construct the histogram.
(2) Figure out the probability density function of predicted price deviation by non-parametric test, denote: $f(\hat{p}_t - p_t)$.
(3) We need figure out the predicted price distribution after the previous step. Defining the predicted

expected price $\hat{P}_{t+k}$ and variance σ^2_{t+k} as the mean and variance parameter, we can calculate the distribution density of the predicted price $f(\hat{p}_{t+1}|p_1, p_2, \cdots, p_t)$. Thus we can obtain the probability of a certain price more or less than predicted price by distribution function.

4. Analysis of Gold Ore Price Distribution

The previous section uses the deviation sample data to estimate the distribution functions of reserves and price. Then, we select the international gold month average prices from 1968 to 2000 to calculate and forecast, and figure out the distribution regularity of gold price. Firstly construct price forecasting model (2) and variance model (3) by the international gold month average prices from 1968 to 2000 as follows:

$$y_t = 0.215y_{t-1} + 0.224y_{t-8} + 0.063y_{t-11} + \hat{u}_t \tag{2}$$

$$\hat{\sigma}^2_t = 0.001 + 0.322\hat{u}^2_{t-1} + \hat{u}^2_{t-2} \tag{3}$$

The model is implemented in the following 3 steps:
Step (1): Predict the price in 2001, $\hat{y}_{2001}$, by the international gold month average prices from 1968 to 2000. Predict the price in 2000 $\hat{y}_{2000}$ by the prices from 1968 to 1999. Predict the price in 1999, $\hat{y}_{1999}$ by the prices from 1968 to 1998.By parity of reasoning, We have forecasted 96 data of eight years, from 1993 to 2000, calculated the deviation between predicted value and real value, and take the deviation data as the sample data.
(2) Rank the sample data in ascending order, divide into groups, calculate the frequency each of group, and draw the histogram.
(3) Whether or not it is normal distribution by χ_2 fitting test or Q-Q plot test. In the figure 1, the

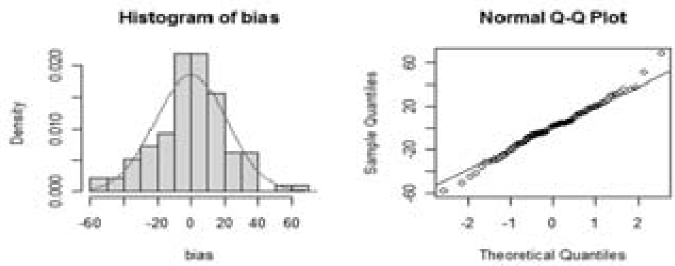

Figure 1: Histogram of predicted deviation from 1993 to 2000

red line of the left is normal curve, the right is normal Q-Q plot. It shows the leptokurtic and fat-tail characteristic. With more trials, we forecast 132 data of eleven years, from 1990 to 2000 as sample data. The deviation histogram is shown in figure 2.

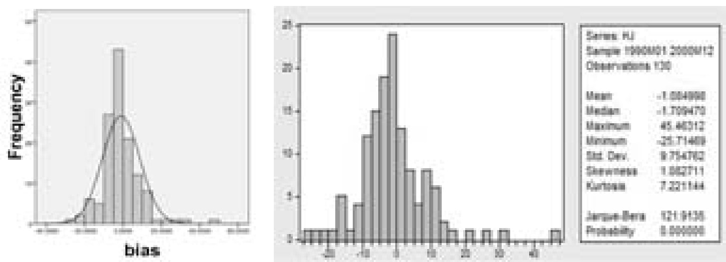

Figure 2: Histogram of predicted deviation from 1990 to 2000 and statistical test

Through the above analysis, we can see that the deviation of international gold predicted price show the leptokurtic and fat-tail characteristic. The normal distribution function failed to illustrate it, the Generalized Error Distribution (GED) is used to describe the above characteristic. GED distribution density function is expressed by:

$$f_{GED}(x) = \frac{\upsilon \cdot exp(-\frac{1}{2}|\frac{x}{\lambda_\upsilon}|^\upsilon)}{\lambda_\upsilon[2^{(1+\upsilon^{-1})}\Gamma(\frac{1}{\upsilon})]} \tag{4}$$

where, υ is the degree of freedom, $0 < \upsilon < \infty$.

$$\lambda_\upsilon = \sqrt{\frac{\Gamma(\frac{1}{\upsilon}) \cdot 2^{-\frac{2}{\upsilon}}}{\Gamma(\frac{3}{\upsilon})}} \tag{5}$$

The distribution is dominated by the parameter, whose variation leads to different distribution. While $\upsilon = 2$, the GED is normal distribution. While $\upsilon < 2$, the GED has thicker tail than normal distribution. While $\upsilon > 2$, the GED has less significant tail than normal distribution. Use the equation 2 and 3 to calculate the parameter mean $\hat{y}_{t+1} = Ey_t$ and variance σ^2 of the t+1 times after obtaining the gold price deviation distribution, like $\hat{y}_{2001} = 277.23$, $\sigma^2 = 0.001$ Viewing them as the mean and variance of distribution function, we acquired the distribution density function of the future price $\hat{y}_{t+1}$.

5. Conclusions

The research of mineral rights investment risk is a very difficult subject. The main reason is that there exist objective uncertainty and the uncertainty coexistence. In addition, the factors in the mineral right market, such as the different forms of market transaction and different investment portfolio, make the issue of mineral right investment risk more complicated and challenging. This paper only focuses on the distribution function of reserves and future price, which is a most fundamental and important issue in the mineral right investment risk research. While the key problem is selecting the random variable data, we ingeniously use the deviation to research the distribution function of reserves and future price. This study establishes the foundation for constructing the mineral right investment risk assessment model, and also has extensive application value.

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