Modeling Dry and Wet Spells of Central Anatolia Region of Turkey

Yıldırım, Fetih
Çankaya University, Department of Industrial Engineering
Öğretmenler cad. No: 14, 100. Yıl
Ankara 06530, Turkey
E-mail: fyildirim@cankaya.edu.tr

Bayrak, Özlem Türker
Çankaya University, Department of Industrial Engineering
Öğretmenler cad. No: 14, 100. Yıl
Ankara 06530, Turkey
E-mail: ozlemt@cankaya.edu.tr

1. Introduction

There is no doubt about the dependency between the amount of rainfall and the crop production yield from the area. Thus, the amount of rainfall is an important factor in many agricultural products. The precipitation area, intensity of rainfall, and the time of it are equally important to be considered in the determination of the statistical distribution of the rainfall.

There are many models suggested for this purpose in the literature. Some of these are as follows: Williams (1952) suggests one parameter model log series distribution whereas Gabriel and Neumann (1957) suggest one parameter model geometric distribution to model dry (wet) spells in Harpenden and Tel Aviv, respectively. Besides one parameter models, many researchers found mixture models suitable for this area. Racsko et al. (1991) and Deni and Jemain (2008) found the mixture of two geometric distributions fits quite well for the dry (wet) spells in Hungary and Peninsular Malaysia, respectively. Dobi-Wantuch et al. (2000) use the mixture of geometric and Poisson distribution for Hungary. Srinivasan (1958) suggests use of mixture of log series and geometric series where the estimation of parameters requires solving quadratic equations. Deni et al. (2009) try mixture of log series distribution with Poisson, truncated Poisson and geometric distribution and introduced two more alternative models named mixture of two log series distributions, and geometric distribution mixed with truncated Poisson distribution to model the data for Peninsular Malaysia.

In this study, the dry (wet) spells, defined as a sequence of dry (wet) days, of Central Anatolia Region of Turkey is considered. This region has scarce water resources resulting in a continental climate where there is an important annual variation in temperature. Winters are usually cold and there seems snow. There is relatively moderate precipitation mostly occurring in summer time. The dry (wet) spells data is modeled by geometric distribution (GD), log series distribution (LSD), mixed two geometric distribution (MGD), mixed two log series distribution (MLSD), mixed log series and geometric distribution (MLGD), mixed log series and Poisson distribution (MLPD), mixed log series and truncated Poisson distribution (MLTPD), and mixed geometric and Poisson distribution (MGPD), and mixed geometric and truncated Poisson distribution (MGTPD) as suggested in the relevant literature discussed above. The probability functions and moment generating functions can be found in Deni et al. (2009). The most relatively appropriate one is selected according to Akaike's Information Criterion (AIC). Some important descriptive statistics of data and their AIC comparisons are given in Section 2. The model fitted to each station's data and concluding remarks can be found in the last section.

2. Data and Comparison of the Selected Models

The data set consists of nine selected rainfall stations in Central Anatolia Region of Turkey for the period of 1975 to 2010. This region is known as one of the mostly agricultural and hydrological activities concentrated region. Therefore, the sizes of the climate and rainfall events are very important for this region. We use the same definition of the wet and dry days and the wet and dry spells as of Deni et al. (2009); i.e., a wet day is defined as a day with a rainfall amount of at least 0.1 mm. (otherwise, it is a dry day), and a wet(dry) spells is a period of consecutive exactly wet(dry) days. Table 1 and Table 2 show the summary of statistics for the data sets of dry and wet spells, respectively, where the statistics are mean, standard deviation, maximum length of dry(wet) spells, probability of dry(wet) day, and the total number of dry(wet) days, respectively.

Table 1.Summary of statistics for the distribution of dry spells at each station in Central Anatolia

	mean	stdev	max	p(d)	Nd
Eregli	7.027948	10.85675	124	0.78409	10310
Beypazarı	5.731953	7.086273	73	0.73671	9687
Aksaray	6.511628	9.583667	107	0.766598	10080
Niğde	6.131546	8.538243	77	0.747966	9835
Sivrihisar	5.445652	6.546292	66	0.723933	9519
Ilgaz	5.274576	8.303886	220	0.710016	9336
Kaman	5.721854	8.371477	129	0.722793	9504
Gemerek	5.446918	7.600876	93	0.725759	9543
Develi	6.058788	8.876824	85	0.752453	9894

Table 2.Summary of statistics for the distribution of wet spells at each station in Central Anatolia

	Mean	stdev	max	p(w)	Nw
Eregli	1.935242	1.334005	11	0.21591	2839
Beypazarı	2.047309	1.409229	13	0.26329	3462
Aksaray	1.982558	1.365028	13	0.233402	3069
Niğde	2.066085	1.466829	13	0.252034	3314
Sivrihisar	2.075472	1.444638	11	0.276067	3630
Ilgaz	2.153021	1.603586	18	0.289984	3813
Kaman	2.194461	1.565476	16	0.277207	3645
Gemerek	2.058219	1.434275	11	0.274241	3606
Develi	1.993264	1.366826	11	0.247547	3255

Table 3 and 4 show the performance measures (AIC values) corresponding to the proposed competing probability models (GD, LSD, MGD, MLSD, MLGD, MLPD, MLTPD, MGPD, MGTPD) for dry spells, and wet spells in each of the selected rainfall stations, respectively. The bold face values indicate that the minimum AIC values which represent the relatively most suitable models at each selected rainfall stations. According to the results shown in Table 3, we see that the relatively most suitable model for dry spells for each station is the mixture of the two geometric distributions. However, according to the results shown in Table 4, we see that the relatively most suitable models are somehow changing; for the Aksaray Station the MGD model, for the Beypazari Station the MGD model, for the Ilgaz Station the MGPD model, for the Sivrihisar and Develi Stations the MGD, MGPD, and MGTPD models are equally, for Kaman Station the MGPD model, for the Eregli Station the MGPD model, for Nigde and Gemerek Stations the MGD, MGPD, and MGTPD models are equally relatively suited models, respectively.

Table 3. Performance Measures (AIC Values) Corresponding to the Proposed Competing Probability Models for Dry Spells in each of the Selected Rainfall Stations.

	GD	LSD	MGD	MLSD	MLGD	MLPD	MLTPD	MGPD	MGTPD
Aksaray	9120.4	8434.9	**8372.5**	8438.9	8434.4	8438.9	8438.9	8524.9	8524.7
Beypazarı	9557.3	8858.0	**8795.7**	8862.0	8819.6	8862.0	8862.0	8900.8	8900.4
Ilgaz	9739.4	8919.4	**8855.0**	8923.4	8910.7	8923.4	8923.4	8996.9	8996.4
Sivrihisar	9720.9	8956.9	**8898.6**	8960.9	8921.1	8960.9	8960.9	9017.6	9015.7
Develi	9403.0	8644.6	**8571.4**	8648.6	8648.1	8648.6	8648.6	8665.6	8664.9
Kaman	9388.8	8588.2	**8531.3**	8590.4	8591.4	8592.2	8592.2	8689.2	8688.4
Ereğli	8851.8	8211.9	**8156.8**	8215.9	8214.2	8215.9	8215.9	8270.3	8269.5
Niğde	9271.4	8564.4	**8507.3**	8568.4	8557.5	8568.4	8568.4	8605.1	8604.1
Gemerek	9744.6	8873.3	**8830.1**	8877.3	8874.3	8877.3	8877.3	8998.3	8996.1

Table 4. Performance Measures (AIC Values) Corresponding to the Proposed Competing Probability Models for Wet Spells in each of the Selected Rainfall Stations.

	GD	LSD	MGD	MLSD	MLGD	MLPD	MLTPD	MGPD	MGTPD
Aksaray	5888.1	4334.2	4257.5	4338.2	4338.2	4338.2	4338.2	**4257.5**	4257.5
Beypazarı	6519.7	4896.7	**4799.4**	4900.7	4900.7	4900.7	4900.7	**4799.4**	**4799.4**
Ilgaz	6973.4	5359.4	5265.3	5363.4	5267.4	5363.4	5363.4	**5264.7**	5264.8
Sivrihisar	6781.6	5135.1	**5028.0**	5139.1	5139.1	5139.1	5139.1	**5028.0**	**5028.0**
Develi	6226.6	4594.9	**4516.9**	4598.9	4598.9	4598.9	4598.9	**4516.9**	**4516.9**
Kaman	6594.4	5145.4	5028.8	5149.4	5149.4	5149.4	5149.4	**5028.3**	5028.8
Ereğli	5522.7	4003.1	3935.7	4007.1	4007.1	4007.1	4007.1	**3935.0**	3935.1
Niğde	6208.8	4672.7	**4594.0**	4676.7	4676.7	4676.7	4676.7	**4594.0**	**4594.0**
Gemerek	6770.2	5087.5	**4997.9**	5091.5	5091.5	5091.5	5091.5	**4997.9**	**4997.9**

3. Parameter Estimation and the Concluding Remarks

The model parameters are estimated by using the maximum likelihood estimation technique. However, since the likelihood functions are nonlinear, they cannot be solved analytically. Thus, the estimates are found by using the "nlminb" subroutine of R language which uses PORT routines for both constraint and unconstraint optimization. Since the parameters of geometric distribution and the weights should be between 0 and 1, the constraint optimization is used. The same is applied to Poisson cases where λ should be positive. Initial values for geometric parameters; p, p1 and p2, are taken as the reciprocal of the mean length of short and long dry(wet) spells; respectively while weight of the mixture models are taken as the proportion of the frequency of short dry(wet) spells relative to the total frequency of both short and long dry(wet) spells. Since Central Anatolia Region of Turkey has a continental climate, the dry(wet) spells can be categorized as short if it is less than or equal to 6 (3), medium if it is between 7 and 19 (4 and 7) and long else. The initial value for λ is taken as the mean length of long dry(wet) spells. The most appropriate model for dry spells was found to be mixture of two geometric distributions. The fitted models for each station and the estimated weights are given in Table 5.

In some stations, it seemed to have 3 possible models for wet spells that could be used according to AIC values (Table 4). However, when the estimated parameters are observed, it was seen that the weight takes value 1 meaning that actually it is not a mixture distribution. Thus, these models are not selected as the appropriate models. The final fitted models can be seen in Table 6. Like the dry spells case, mixture of two geometric distributions gave quite good approximations in most stations. However, in others, mixture of a geometric distribution with a Poisson distribution gave better approximations than the rest.

Table 5. Fitted Models for Dry Spells at Each Station

	best model	w	p_1	p_2
Aksaray	MGD	0.76	0.27	0.06
Beypazarı	MGD	0.58	0.34	0.10
Ilgaz	MGD	0.72	0.31	0.09
Sivrihisar	MGD	0.53	0.39	0.12
Develi	MGD	0.83	0.26	0.06
Kaman	MGD	0.76	0.30	0.08
Ereğli	MGD	0.84	0.22	0.05
Niğde	MGD	0.75	0.27	0.07
Gemerek	MGD	0.74	0.31	0.08

Table 6. Fitted Models for Wet Spells at Each Station

	best model	w	p_1	p_2	p	λ
Aksaray	MGPD	0.9999			0.5046	10.0461
Beypazarı	MGD	0.5472	0.4886	0.4886		
Ilgaz	MGPD	0.9989			0.4676	13.6983
Sivrihisar	MGD	0.4667	0.4821	0.4821		
Develi	MGD	0.5925	0.5015	0.5015		
Kaman	MGPD	0.9995			0.4569	13.5818
Ereğli	MGPD	0.9972			0.5215	7.1738
Niğde	MGD	0.7029	0.4839	0.4839		
Gemerek	MGD	0.7149	0.4859	0.4859		

As a result, it is found that mixture models give better approximations than single models for dry(wet) spells for Central Anatolia Region of Turkey which is similar to results found by Deni et al. (2009). This type of studies would be beneficial for developing many management policies. As a future study, it will be extended to all stations in that area so that the whole picture to develop policies can be seen for that region. Also it could be extended to other regions of Turkey so that the interactions between the regions that share the same water resources could be handled.

REFERENCES (RÉFERENCES)

Deni, S.M., and Jemain, A.A. (2008), Probability models for dry spells in Peninsular Malaysia, Asia-Pasific, J. Atmos. Sci. 44(1), 37-47.

Deni, S.M., Jemain, A.A., and Ibrahim, K. (2009), Mixed probability models for dry and wet spells, Statistical Methodology 6, 290-303.

Dobi-Wantuch, I., Mika, J., and Szeidl, L. (2000), Modeling wet and dry spells with mixture distributions, Meteorol. Atmos. Phys. 73, 245-256.

Gabriel, K.R., and Neumann, J. (1957), On the distribution of weather cycles by length, Quart. J. Roy. Meteor. Soc. 83, 375-379.

Racsko, P., Szeidl, L., and Semenov, M. (1991), A serial approach to local stochastic weather models, Ecol. Model. 57, 27-41.

Srinivasan, T.R. (1958), A statistical model suitable for describing weather persistence, Indian J. Meteorol. Geophys. 10, 321-324.

Williams, C.B. (1952), Sequences of wet and of dry days considered in relation to the logarithmic series, Quart. J. Roy. Meteor. Soc. 78, 511-516.

RÉSUMÉ (ABSTRACT)

Water resources are getting scarce in each year. Thus, water resource management is becoming more and more crucial, especially for Turkey since it is nowadays a water stress country having, approximately 1,500 m3 water per capita. It is estimated by Turkish Statistical Institute that it will be about 1,000 m3 by the year 2030. Most of the water is used by the irrigation sector in Turkey. Therefore, to determine an appropriate rainfall distribution model with minimum number of estimated parameters is an essential step for the water resource management. In this study, the dry (wet) spells, defined as a sequence of dry (wet) days, of selected 9 stations in Central Anatolia Region of Turkey between 1975-2010 are considered. The dry (wet) spells' data is modeled by 9 different probability models named as geometric distribution (GD), log series distribution (LSD), mixed two geometric distribution (MGD), mixed two log series distribution (MLSD), mixed log series and geometric distribution (MLGD), mixed log series and Poisson distribution (MLPD), mixed log series and truncated Poisson distribution (MLTPD), and mixed geometric and Poisson distribution (MGPD), and mixed geometric and truncated Poisson distribution (MGTPD) as the literature (Dobi-Wantuch et al., 2000; Denia et al., 2009 etc) .The fitted models are compared on the basis of Akaike's Information Criteria (AIC). It is found that the dry spells can be better approximated by a mixture of two geometric distributions in all stations whereas the wet spells are better modeled by either mixture of two geometric distributions or a mixture of geometric with a Poisson distribution. In all cases, it is seen that the weight of the first distribution is highly larger than the second one.

Keywords: rainfall, dry and wet spells, mixture models, water resource management, Akaike's Information Criterion.