In service elementary school teachers' conceptions of arithmetic mean

Chatzivasileiou, Evanthis
Aristotle University of Thessaloniki, Faculty of Education
1 Anatolikis Thrakis Street
Thessaloniki (54453), Greece
evanthys@otenet.gr

Michalis, Ioannis
Aristotle University of Thessaloniki, Faculty of Education
14 Archilochou Street
Thessaloniki (54352), Greece
jonious@otenet.gr

Tsaliki, Christina
Aristotle University of Thessaloniki, Faculty of Education
10 Nikis street
Thessaloniki (55535), Greece
xtsaliki@sch.gr

Sakellariou, Io
Aristotle University of Thessaloniki, Faculty of Education
14 Archilochou Street
Thessaloniki (54352), Greece
iosakel@hotmail.com

People today are inundated daily with data and information through various sources of information. Such information is often contradictory, conflicting or confused. Therefore everyone should have the ability to understand information on the actual dimension, as determined by the collection, organization, analysis and interpretation, and to assess the reliability and the quality (Μηρώι εο, 2006). In other words, they should be able to manage and evaluate them, to reach decisions relevant to their lives. To obtain what Friel & Joyner (1997) refer to as "sense of data ".

The statistics, as a tool for transforming the raw information into highly structured data (Carvalho & Cesar, 2002), allows us to understand the reality and make decisions as a result of their interpretation. Consequently basic statistical knowledge is necessary so that people are able to take informed decisions on issues that concern them (Lajoie et al, 1995), developing critical thinking and sense of autonomy. The development of statistical literacy goes to primary education during which students are introduced to statistical concepts.

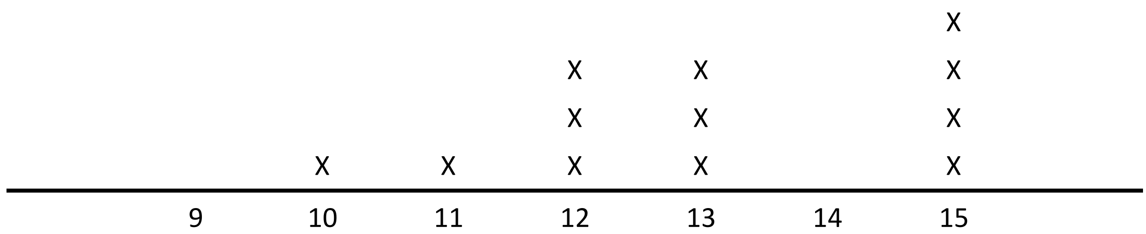
The purpose of this study is to determine whether there is an understanding of the concept of the arithmetic average by elementary school teachers as described by Strauss & Bicher (1988)-as the knowledge of the seven properties. These properties are fundamental and tap three aspects of the concept: The statistical aspect (the first tree properties), the abstract aspect (next three properties) and the central aspect (the last property). Understanding the concept of arithmetic mean through the development of its properties is important because knowing how to calculate the average does not imply its comprehension (Batanero et al, 1994). Specifically, the objectives of the research focused on measuring the extent of research the subjects are aware of these properties, focusing on properties in accordance with the classification of Goodchild (1988), corresponding to measure of location and the representative value. Are the teachers able to manage the data in a dataset when there are between these extremes? Do they understand the concept of the average as a point of balance of the dataset? Do they recognize the idea of representing a set of values to a value that does not have relevance to physical reality and cannot be displayed on the data values?

The properties which were primarily investigated were: (a) The average is located between the extreme values, (b) the sum of the deviation from the average is zero, (c) when one calculates the average, a value of zero, if its appears, must be taken into account & (d) the average value is representative of the values that were averaged. The first two properties deal with idea of interpretation of the mean as a measure of location and the second two address a representative interpretation of the mean (Leon & Zawojewski, 1991).

Our research was conducted in December 2010 in a sample of 130 in service elementary school teachers. For data collection form were used questionnaires distributed and collected by the presence of the researchers

To investigate the comprehension of the average as a measure of location and the knowledge of the property "the average is located between the extreme values" (Strauss & Bichler, 1988, 66) we used an example similar to the "Potato Chips Problem "of Mokros & Russell (1995, 11). We asked teachers to raise prices in a price list to show a specific average. According to the views of Mokros & Rusell (1995), working from the average towards data is more difficult than ordinary school problems in which we calculate the average and requires an understanding of the relationship between the data and the average. 70% of teachers responded correctly, giving values corresponding to the infused average, while 30% did not answer or gave the wrong answer. Of the teachers who responded, only one placed the average value to the maximum allocation confusing it with the maximum value, and none placed it outside the extreme values. The responses of teachers resulting in what is related to the property under investigation, they are all aware of and able to apply this property to a reverse process of construction the data.

Understanding the average as a point of balance of the data reflects an understanding of the second property and is the basis of the definition of standard deviation ("the sum of the deviations from the average is zero", Strauss & Bichler, 1988, 66). To study the comprehension of the average as a point of balance a line chart was presented to the subjects to which data was balanced on a value corresponding to the average asking teachers to assess if the addition of two extra price -4 and +2 deviation from the average would shift the measure and in what direction (figure 1).



Age in years
(fig. 1) Children's age

The teachers estimated that: 53% the average will decrease (right), 37% average will remain stable, 8% average will increase and 2% didn't respond. Only half teachers answered correctly to the problem presented to them, that since the sum of the deviations is negative (2-4 =- 2) therefore and in order to balance the distribution, the mean will shift to a lower position. This indicates the difficulties in understanding the average as the point around which a data distribution is balanced.

Characteristic property of the representative dimension of the average is the phrase: "when one calculates the average, a value of zero, if it appears, must be taken into account" (Strauss & Bichler, 1988, 66). The difficulty is this case is that the subjects who have not developed a statistical understanding of the term, but merely to apply the computational algorithm, in the process of addition, zero values are ignored. To study the comprehension of the statistical dimensions of the average as representative of all the values including zero, which in an algorithmic sequence will not play a role, we gave to the subjects a set of 5 values and a specific average and asked them to predict what would happen if we added an extra zero value. 62% of respondents answered correctly, that the average would be smaller but significant proportion of almost 26% replied that the average would remain constant. It is also important the percentage of teachers who declared unable to predict the consequence of adding a zero value in all prices. Strauss & Bichler (1988) argued that this property is difficult to be absorbed by students 12 to 14 years. Our research implies that this difficulty occurs with a powerful way also in elementary school in service teachers, which is interpreted as weakness of conceptual understanding of average as a measure that summarizes the data and is representative of the values (Leon & Zawojewski, 1991).

The knowledge of the use of an appropriate measure of central tendency and the ability of teachers to manage it, in a real problem situation, to exploit the information from a data set, is a condition of the statistical understanding of them (Χαηδεπαληαή ο, 1998). The statistical dimension of the average resulting from the application of the last property, as representative of values from which it originates. To study this property we applied a variant of a Sorto's (2004, 128) problem in which the values recorded by repeated measurements to arrive at a value that describes the reality, with the utmost precision. Among the values was presented an extreme value, which according to Χι νπβεξάθεο (2009, 100) was far more than 1,5 IQR (figure 2).

35g	34,3g	40g	35g	33,95g	34,2g	100g	30g	33g

(Fig. 2) Weight measurements

Teachers were asked to choose the method that they considered to give the more accurate result of measurement. The largest percentage (33%) of teachers selected the arithmetic mean, excluding the outlier, which was the correct answer. Almost 25% of teachers indicated the arithmetic mean of all values, 16% the mode, 13% the value had more decimal digits (the more detailed value), 9% the median (table 1).

Method	Answers	Answers in Sorto's research
The mode	16%	12%
The more detailed value	13%	19%
The median	9%	-
The arithmetic mean	25%	19%
The arithmetic mean without the extreme value	33%	26%
Other	4%	24%

(Table 1) Preferred method

In Sorto's (2004) study corresponding answers were given by teachers, as shown in Table 1, which is estimated as a weakness in choosing the appropriate measure of central tendency because of the difficulty in understanding the statistical dimension of the measure.

Attempting a comprehensive assessment of the perception of the average properties (Strauss & Bichler, 1988), we set to teachers proposals relevant to Strauss' and Bichler's (1988) properties of the average to be selected true - false. The responses received are shown in Table 2.

Proposals	True	False	Strauss & Bichler Property	Wright answer
When one calculates the average a value of zero, if it is appears, must not be taken into account	20%	**75%**	F	The false
The average does not necessarily equal one of the values that was summed	**78%**	18%	D	The true
The sum of the deviation from the average is zero	**31%**	61%	B	The true
The average value is representative of the values that were averaged	**74%**	22%	G	The true
If a set of values add a value smaller than the average the average is not affected	8%	**88%**	C	The false
An extreme value must be excluded when the average is calculated	21%	**74%**	C,A	The false
The arithmetic mean is the same with the median	22%	**70%**	G	The false
The average is the point of balance of the deviation	**69%**	24%	B	The true
The average separates the deviation into two halves	19%	**76%**	G	The false

(Table 2) The properties of the arithmetic mean- Strauss & Bichler, 1988

The replies indicate that teachers are familiar with the terminology of statistical measures, but have significant difficulties in using them to solve problems of everyday life. However significant number of teachers equal to ¼ and more presents considerable difficulties in understanding the basic properties and concepts of descriptive statistics, although mathematics taught students in fifth and sixth grade of primary school concern such concepts.

The research shows that although for most teachers is relatively easy to calculate the average by applying the computational algorithm, is quite difficult to understand the measure as a representative value that summarizes the data and need not be a real number of the dataset. It is a central value around which a balances a data distribution. In our view, understanding the concept of average goes through understanding the concept of data distribution.

Subtitle

The properties of the arithmetic mean

Figure or Table Title

(Fig. 1) Children's age. (Fig. 2) Weight measurements. (Table 1) Preferred method. (Table 2) The properties of the arithmetic mean.

REFERENCES (RÉFERENCES)

Batanero, C., Godino, J.D., Green, D.R., Holmes, P. & Vallecillos, A. (1994), Errors and difficulties in understanding elementary statistical concepts. International Journal of Mathematics Education in Science and Technology, 25(4), 527-547

Carvalho, C. & Cèsar, M. (2002). Sharing Ideas and Statistics Learning: The Role of Peer Interactions in School Context. Proceedings of the 6th International Conference of Teaching Statistics. Durban: ISI/IASE.

Goodchild, S. (1988). School Pupils' Understanding of Average. Teaching Statistics, 10, 77-81

Friel, S.N. & Joyner, J.M. (Eds.). (1997). Teach–stat for Teachers: Professional development manual. Palo Alto, CA: Dale Seymour

Lajoie, S., Jacobs, V. & Lavigne, N. (1995). Empowering children in the use of Statistics. Journal of Mathematical Behavior, 14, 401-425

Leon, R. & Zawojewski, J.S. (1991). Use of the Arithmetic Mean: An Investigation of four Properties, Issues and Preliminary Results. Paper presented at the 3rd International Conference on Teaching Statistics: Vol 1. School and General Issues. The Netherlands: Voorburg

Mokros, J. & Russell, S.J. (1995). Children's concepts of average and representativeness. Journal for Research in Mathematics Education, 26, 20-39

Sorto, M. A. (2004). Prospective Middle School Teachers' Knowledge about Data Analysis and its Application to Teaching. Doctoral dissertation, Michigan State University, East Lansing

Strauss, S. & Bichler, E. (1988). The Development of Children's Concepts of the Arithmetic Average. Journal for Research in Mathematics Education, 19, 64-80

Μηράιεο, Ι. (2006). Τν Αλαιπηηθό πξόγξακκα ζπνπδώλ ηεο Σηαηηζηηθήο ζην Δεκνηηθό ζρνιείν θαη νη αληη εξκελεία θαηθαηαλόεζε – ησλ καζεηώλ/ηξηώλ ησλ ηει επηαίσλ ηάμεσλ γηα ηηρ γξαθηθέο απεηθνλίζεηο ησλ δεδνκέλσ δηαηξιβή, Θεζζαινλίθε: Αξηζηνηέιεην Παλεπηζηήκην Θεζζαινλίθεο (ΑΠΘ)

Χαηδεπαληειήο, Θ. (1998). Σηαηηζηηθή ζηελ εθπαίδεπζε. Θεζζαινλίθε: ΑΠΘ, Παηδαγσγηθό Τκήκα Δεκνηηθήο Εθπαίδεπζεο

Χινπβεξάθεο, Γ. (2009), Εηζαγσγή ζηε ζηαηηζηηθή, Ειιεληθά γξάκκαηα, Αζήλα

RÉSUMÉ (ABSTRACT) — optional

The arithmetic mean is not only a basic concept of statistics and experimental science but it is the most frequently used concept in everyday life (Pollatsek et al, 1981, Leon, R.M. & Zawojewski, S.J, 1991). The concept of average is related to the comprehension of seven properties of the arithmetic mean (Strauss & Bichler, 1988).

This paper aims to explore the in service elementary school teachers' conceptions of the arithmetic mean. The research took place in December 2010 and investigates the conceptions of the seven properties of the arithmetic mean. A paper & pencil questionnaire has been designed, comprised of simple questions which were undertaken by 130 Greek in service elementary school teachers' of Thessaloniki region in Northern Greece. The questionnaires were undertaken in a collective way.

The results show a lack of understanding of some properties like: (a) the sum of deviation from the average is zero, (b) the average does not necessarily coincide with one of the values which are composed by it, (c) the average is a representative value of the data. Another difficulty concerns misunderstanding of the distribution of a data set.

The data analysis revealed that the arithmetic mean is quite complicated to be fully understood (MacCullough, 2007) by the in service teachers even though it seemed that they used it in several occasions