One-sided EWMA control chart for monitoring high yield processes

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A one-sided Exponential Weighted Moving Average (EWMA) control chart is introduced to monitor the fraction p of nonconforming products in high yield processes. It is designed to detect upward shifts of p screening the non-transformed geometric counts i.e. the number of conforming products between two nonconforming ones. Its algorithmic function is theoretically established and numerous performance measures are extracted using simulating methods. The efficiency and the performance superiority of the proposed one-sided EWMA control chart is verified through comparisons with a recently developed high yield chart, the simple two-sided EWMA control chart. Additionally, an algorithm of optimally designing the chart is developed and an optimality table for the most usual parameter values is adduced.

Introduction

Automation has prevailed in the modern industry leading inevitably to the emergence of high yield processes. Examples of such processes can be easily found in the semiconductor and telecommunications industries where the fraction p of nonconforming products is usually on the order of parts -per-million (ppm). The traditional control charts, fail to detect in time excessive out-of-control shifts at these processes and as Goh (1987) showed, they usually result in inappropriately high false alarm rates. The demand to effectively monitor the increasingly many high yield processes led to the development of control charts based on the geometric counts X, the count of products inspected until a nonconforming product is encountered.

The geometric counts were firstly studied by Calvin (1987) and Nelson (1994) followed proposing a control chart based on transforming the geometric count X to $X^{1/3}$. Similar transformations of X were studied by numerous other researchers. Recently Chang and Gan (2001) used the CUSUM statistic applying it on the non-transformed observations obtained from geometric, binomial and Bernoulli distribution. They concluded that the CUSUM control chart performs efficiently even for very small values of p.

The latest evolution was achieved due to Yeh et al. (2008) who implemented the EWMA control chart on geometric counts and demonstrated its superiority in detecting upward shifts in p (in comparison with the aforementioned CUSUM) based on simulation results.

In this paper the work of Yeh et al. (2008) is extended and refined in an attempt to obtain increased detection sensitivity. Following the technique of Shu et al. (2007), a new one-sided EWMA control chart is developed for monitoring the upward shifts of the fraction of nonconforming products in a high-yield process. Shu et al. (2007) were the first to conclude (studying normally distributed data), that the one-sided EWMA chart lacks the serious inertia feature characterizing the traditional two-sided EWMA charts especially when the EWMA statistic is far below the target before the occurrence of a shift.

The basic idea on which the one-sided EWMA chart is based, is the accumulation of positive (or negative) deviations from the target only and the truncation of the negative (or positive, correspondingly) deviations from the target to zero in the computation of each EWMA statistic. It should be noted that the use of the one-sided control chart assumes a known shift direction. In the context of the present paper, only the increasing shifts in the fraction of nonconforming products (from p_0 to $p > p_0$) are examined which lead to decreasing shifts of the geometric count X. Thus, the proposed control chart will be called hereafter *lower geometric EWMA*. Regarding the chart's design, the maintenance of a specified false alarm rate is ensured through appropriate adjustment of the chart's control limits. The definition of these control limits, the examination of the truncated geometric variable X and the theoretical foundation of the one-sided EWMA statistic are presented in section 2. In section 3 the optimal design parameters of the new control chart are extracted and its performance is investigated with the help of appropriate comparisons with the recently developed two-sided EWMA control, chart of Yeh et al. (2008).

One-sided EWMA control charts

In this section the work of Shu et al. (2007) is extended to the case of geometrically distributed data and the lower geometric EWMA control chart is theoretically founded. Consider an attribute process where each product can be conforming or nonconforming. Let X denote the number of products monitored until a nonconforming one is encountered. If the probability of each product be nonconforming is p, then the distribution of X is geometric with parameter p i.e. with probability function $P(X = x) = (1 - p)^{x-1}p$, $x = 1, 2, ...$ In this case the expected value and the variance of X, are $E(X) = \mu = 1/p$ and $Var(X) = \sigma_X^2 = (1 - p)/p^2$ correspondingly.

A high yield production process is considered to be in-control with parameter p_0 when there are only random causes for its underlying variability. In this case the values X_t of the variable X randomly fluctuate around their mean value $E(X_t) = \mu_0$. When there are significant external, assignable causes the process is considered to be out-of-control at a quality level p. In that case the deviation of X_t from μ_0 is considered significant and action should be taken to secure the proper function of the process.

As it is already mentioned, in this paper only the non-random increases in the fraction of nonconforming products are examined leading the geometric counts X_t to decrease. Following the same methodology developed by Shu et al. (2007), only the observations X_t below the mean μ_0 are accumulated. Each geometric count X_t is transformed into

$$(1) \qquad X_t^- = min\{\mu_0, X_t\} = \mu_0 - max\{0, \mu_0 - X_t\}$$

It is easily extracted that the mean of the new variable X_t^- is

$$(2) \qquad E(X_t^-) = \mu_0 - \sum_{t=0}^{\mu_0}(\mu_0 - X_t)f(\mu_0 - X_t) = \frac{1 - 2p - (1 - p)^{\frac{1+p}{p}}}{p(1 - p)}$$

where $f(\mu_0 - X_t)$ is the probability density function (pdf) of $\mu_0 - X_t$.
The quantity $E(X_t^{-2})$ is given by the following formula

$$(3) \qquad \begin{aligned} E(X_t^{-2}) &= E(\mu_0^2 + max^2\{0, \mu_0 - X_t\} - 2\mu_0 max\{0, \mu_0 - X_t\}) \\ &= \frac{p^2 - 4p + 2 + (1 - p)^{1/p}(-p^2 + 5p - 4)}{p^2(1 - p)} \end{aligned}$$

Then the variance of the X_t^- is calculated considering equations (2) and (3)

$$(4) \qquad Var(X_t^-) = \sigma_{X_t^-}^2 = E(X_t^{-2}) - E^2(X_t^{-2})$$

The traditional EWMA control statistic for $t = 1, 2, ...,$ is given by the following formula

$$G_t^- = \lambda X_t^- + (1-\lambda) G_{t-1}^- \quad (5)$$

where $0 < \lambda < 1$ is a pre-determined smoothing parameter and $G_0 = \mu_0$. It can be proved that the EWMA statistic can also be written

$$G_t^- = \sum_{j=0}^{t-1} \lambda (1-\lambda)^j X_{t-j}^- + (1-\lambda)^t G_0 \quad (6)$$

From (6) it is deduced that the mean and the variance of the EWMA statistic are respectively (see Montgomery, 2005)

$$E(G_t^-) = E(X_t^-) \sum_{j=0}^{t-1} \lambda (1-\lambda)^j = E(X_t^-)(1-(1-\lambda)^t) + (1-\lambda)^t \mu_0 \quad (7)$$

and

$$Var(G_t^-) = Var(X_t^-) \sum_{j=0}^{t-1} \lambda^2 (1-\lambda)^{2j} = \sigma_{X_t^-}^2 \frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2t}) \quad (8)$$

Then, according to the EWMA control chart technique, the lower control limit (LCL) below which a signal is produced is given by the formula replacing (7) and (8)

$$LCL = E(G_t^-) - L\sqrt{Var(G_t^-)} \quad (9)$$

where L is a pre-determined constant multiplier used to adjust (in combination with λ) the in-control performance of the EWMA control chart. Note that the term $(1-\lambda)^{2t}$ involved in equations (7)-(9) approaches zero as t gets larger. Thus, after several time periods have passed, the LCL will approach a steady-state value (see Montgomery, 2005) given by

$$LCL = E(X_t^-) - L\sqrt{\frac{\lambda}{2-\lambda}} \sigma_{X_t^-} \quad (10)$$

For reasons of simplicity, the limiting form of the term $(1-\lambda)^{2t}$ will be considered equal to zero throughout this paper.

Performance evaluation and comparison

In this paragraph the performance of the proposed control chart is examined and compared to the two-sided EWMA control chart developed by Yeh (2008). The efficiency of the control charts is traditionally measured with the average run length (ARL) i.e. the number of Xs until a shift is detected. However, in this paper where Xs represent geometric counts, the ARL does not comprehensively explain in what product the shift is detected. Thus, the alternative performance measure of ANIS (average number of items until shift) is used. Two types of ANIS can be considered: the ANIS_0 describing the average number of items until a shift is detected when the process is in-control, and the ANIS_1 declaring the average number of items intervened from the item where the shift has occurred until the item where it is detected when the process is considered out-of-control. For both forms of ANIS the following relationship with the ARL evidently holds

$$(11) \qquad ANIS = \mathrm{E}\{\text{number of } X\} \cdot \mathrm{E}\{\text{length of } X\} = ARL \cdot \frac{1}{p}$$

Note that throughout this paper only zero-state scenarios are considered meaning that when the monitoring begins, the process is considered to be exactly in the center of the in-control region when ANIS_0 is concerned and in an out-of-control state when ANIS_1 is concerned. Both forms of ANIS are calculated through monte carlo simulations involving 250,000 replications through an optimization procedure: firstly, the desired ANIS_0, the smoothing parameter λ and the shift level that is interested to be quickly detected, are specified. Then the parameter L is appropriately chosen to minimize the ANIS_1 at the specified shift while in the same time satisfies the ANIS_0. This optimization design was implemented both on the lower and the two-sided EWMA control chart of Yeh et al. (2008) and some representative results are cited in table 1.

More specifically, table 1 presents the out-of-control ANIS_1 of the two control charts when the in-control ANIS_0 is 50,000 and 70,000. The initially in-control values of p_0 take the values 0.0001, 0.0002 and 0.0003 and various upward shifts are examined. Two different values of the smoothing parameter λ were chosen, $\lambda = 0.1$ and 0.5. For each chart and for given λ and ANIS_0, the optimum parameter L is calculated to minimize the ANIS_0 considering a shift at $p = 0.0005$.

A close examination of the table 1 reveals a similar optimality behavior of the two charts for both examined values of λ. More specifically, the optimal L slightly increases with the in-control fraction p_0 of non-conforming products. For example, for the case of $\lambda = 0.1$, when the $\text{ANIS}_0 = 70,000$, the optimum value of L for the detection of a shift from $p_0 = 0.0001$ to $p = 0.0002$ (100% increase in quality level) is 0.214 for the lower EWMA control chart and 0.250 for the two-sided chart. On the other hand the optimum value of L for the detection of a shift from $p_0 = 0.0003$ to $p = 0.0006$ (also 100% increase in quality level) is 0.806 and 0.765 correspondingly. The same holds for the case of $\lambda = 0.5$.

Table 1 also facilitates the immediate comparison of the two control charts for the detection of the same shifts and under their optimum parameter λ. For every examined shift the lower geometric EWMA control chart is more sensitive as its ANIS_1 is by far smaller than the corresponding ANIS_1 of the two-sided EWMA control chart.

Conclusion

The evolution of the modern manufacturing techniques has led inevitably to the emergence of increasingly many high yield processes and the demand for the development of extra sensitive control charts. In this attempt a new one-sided EWMA control chart is proposed for monitoring geometric counts. The comparison of the new control chart with the older two sided EWMA control proposed by Yeh et al. (2008) chart reveals its superior sensitivity. Undoubtedly, extended research could be conducted in the future on the proposed control chart. For example, the behavior of the chart could be tested under the scenario of fast initial response (FIR). Moreover more work has to be done to extract its performance measures with analytical techniques like Shu et al. (2007) proposed based on Markov modelling.

		Zero-state $ANIS_0$							
		$\lambda = 0.1$				$\lambda = 0.5$			
		70000		50000		70000		50000	
p_0	p	$EWMA_l$ L=0.214	$EWMA_t$ L=0.250	$EWMA_l$ L=0.072	$EWMA_t$ L=0.151	$EWMA_l$ L=1.010	$EWMA_t$ L=0.753	$EWMA_l$ L=0.740	$EWMA_t$ L=0.616
0.0001	0.0002	8735	8840	8110	7400	9418	13665	8693	10332
	0.0003	4103	4517	4170	3933	4384	6999	4206	5272
	0.0004	2803	2994	2751	2703	2884	4331	2821	3436
	0.0005	2102	2246	2080	2076	2128	3157	2094	2491
p_0	p	L=0.555	L=0.559	L=0.403	L=0.401	L=1.384	L=1.000	L=1.201	L= 0.850
0.0002	0.0003	12682	16928	10962	12336	12670	23044	10862	16052
	0.0004	5561	8584	4921	6677	5236	11569	5055	8545
	0.0005	3273	5708	3038	4506	3184	7514	3098	8726
	0.0006	2269	4319	2251	3393	2293	5317	2267	4270
p_0	p	L=0.806	L=0.765	L=0.657	L=0.609	L=1.601	L=1.103	L=1.429	L=1.004
0.0003	0.0004	17713	22353	14608	16924	15926	28976	12743	21016
	0.0005	7298	11364	6520	8804	6864	14648	5718	11488
	0.0006	4324	7940	3907	6295	3867	9734	3787	7803
	0.0007	2954	7901	2660	4605	2635	6943	2524	5462

Optimum L and the achieved $ANIS_1$ for the lower geometric EWMA ($EWMA_l$) and the two-sided EWMA ($EWMA_t$) control chart

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