Modelling electricity spot and forward prices by ambit fields

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Motivation

Electricity markets have been liberalised worldwide in the last two decades. The main products traded on such markets are the spot price, forwards and futures and options written on them. The questions we wish to address here is how such electricity prices can be modelled mathematically. A new model class for electricity spot and forward processes should be able to account for the most important stylised facts of electricity markets, such as strong seasonal patterns, very pronounced volatility clusters, extreme spikes and jumps and the Samuelson effect, see [5]. Note that the Samuelson effect refers to the finding that the volatility of the forward price tends to increase when time to maturity/delivery approaches zero.

In the following we will review a new modelling framework for electricity prices based on *ambit fields*, which has recently been developed in [1, 2, 3].

Ambit fields

An ambit field is a random field where we impose the following structural assumptions. We denote by $t \in \mathbb{R}$ the (current) time and by $x \in \mathbb{R}^n$ for $n \in \mathbb{N}$ the spatial dimension, which we will set to $n \in \{0, 1\}$ in the following. Then, we define an ambit field by

$$(1) \quad Y_t(x) = \mu + \int_{A_t(x)} g(\xi, s; x, t)\, \sigma_s(\xi)\, L(\mathrm{d}\xi, \mathrm{d}s) + \int_{D_t(x)} q(\xi, s; x, t)\, a_s(\xi)\, \mathrm{d}\xi \mathrm{d}s,$$

where $\mu \in \mathbb{R}$ is a constant, $A_t(x)$, and $D_t(x)$ are ambit sets, g and q are deterministic function, $\sigma \geq 0$ is a stochastic field referred to as the *volatility*, and L is a *Lévy basis*, see e.g. [4]. Clearly, we need some integrability assumptions to ensure that the integrals above are well defined, see [2, 3].

Modelling energy spot prices by Lévy semistationary processes

First of all, we present a model for electricity spot prices, which is based on the null–spatial case (i.e. $n = 0$) of an ambit field: a *Lévy semistationary* $\mathcal{LSS}$ process, see [2]. An arithmetic model for the electricity spot price, denoted by $S = (S_t)_{t \geq 0}$, is given by

$$(2) \quad S_t = \Lambda(t) + Y_t,$$

where $\Lambda : \mathbb{R}_+ \to \mathbb{R}_+$ denotes a deterministic seasonal function and

$$(3) \quad Y_t = \int_{-\infty}^{t} g(t-s)\omega_{s-} dL_s + \int_{-\infty}^{t} q(t-s) a_s ds,$$

for damping functions $g, q : \mathbb{R}_+ \to \mathbb{R}_+$ and for càdlàg, positive, stationary processes $\omega = (\omega_t)_{t \in \mathbb{R}}$ and $a = (a_t)_{t \in \mathbb{R}}$ which are independent of the two–sided Lévy process $L = (L_t)_{t \in \mathbb{R}}$.

Note that an $\mathcal{LSS}$ process is stationary as soon as the volatility components ω, a are stationary processes.

The $\mathcal{LSS}$–framework for modelling electricity spot prices is build on four main characteristics: First, deseasonalised prices are modelled directly in stationarity to reflect the fact that spot prices are equilibrium prices matching supply and demand on the market. Hence, we do expect them to be in a stationarity. Further, through the general kernel function g, we have a great flexibility in modelling the autocorrelation function of the spot price (which is directly determined by the kernel function) and in modelling the Samuelson effect. Also, given that we have a general Lévy process as the driving process, our model allows for jumps and spikes. Lastly, the stochastic volatility component ω reflects the volatility clusters often found in financial data.

For further details on $\mathcal{LSS}$–based models for electricity spot prices we refer to [2] and focus now on models for electricity forward contracts based on ambit fields.

Modelling electricity forward prices by ambit fields

Here, we do not stay in the null–spatial case, but allow for a spatial component (i.e. $n = 1$), which will denote the time to delivery of a forward contract. In [3], we propose to use a special case of an ambit field given by

$$(4) \quad f_t(x) = \int_{A_t(x)} g(\xi, t-s; x)\sigma_s(\xi)L(d\xi, ds),$$

for modelling the forward price of electricity. Here, $t \geq 0$ denotes the current time, $T > 0$ denotes the time where the delivery starts and $x = T - t$ denotes the corresponding time to delivery. In order to fully specify the model, we also have to specify the ambit set $A_t(x)$, the damping function g and the (stationary) stochastic volatility field $\sigma_s(\xi)$. For analytical tractability, we typically assume that σ is independent of L, and in order to ensure that $f_t(x)$ is stationary in time t, we take the ambit sets to be of the form $A_t(x) = A_0(x) + (0, t)$.

Outlook

Ambit fields and their null–spatial case – Lévy semistationary processes – form an integrated framework for modelling both electricity spot and forward prices simultaneously. Due to their general structure, they are able to account for the various stylised facts of electricity prices, see also [2] for first empirical studies. Finally, since ambit fields are highly analytically tractable, we can easily derive the corresponding forward and option prices.

References

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