The Analysis of Czech Macroeconomic Time Series (L'analyse des séries temporelles macroéconomiques tchèque)

Marek, Lubos
Department of Statistics and Probability, University of Economics, Prague
W. Churchill sq. 4
Prague, 130 67, Czech Republic
E-mail: marek@vse.cz

Introduction

In this paper, we will analyze the time series of Czech Republic exports and imports. The paper is focused on applications; its goal is to identify models suitable for the time series of Czech Republic exports (import) and utilization of such models for predictions.

Data of the Czech Republic foreign trade time series are published monthly by the Czech Statistical Office. However, the published data are not actual – at the first stage, only estimates are published, which are subsequently made more accurate (in several steps). Moreover, even these estimates are published with a time delay of two to three months. For example, at the time of writing this paper (May 2011), the values of the above-mentioned time series are only known until February 2011. On the other hand, values of certain series (such as the exchange rates of CZK with respect to other currencies) are published more or less immediately and are exact. Hence, if we are able to identify a model describing the dependence between the above-mentioned time series, such a model can, at least, considerably speed up construction of preliminary estimates, thus significantly reducing the waiting time for one of the most important and most closely watched macroeconomics time series in the Czech Republic.

In this paper, we will analyze the time series of exports (imports) from the Czech Republic as a whole. Then we will try to prove its mutual dependence with the CZK/USD (or CZK/EUR) exchange rate time series. A suitable model will be sought by utilizing the theory of stochastic models for time series [1], [2], [3], [4], [7], as well as the theory of transfer function models [5], [6]. Our effort will be focused on construing predictions of the CR export time series for several future periods.

The source of our data is the Czech Statistical Office (time series of Czech Republic exports, expressed in million CZK in fixed prices) and the Czech National Bank (time series of the CZK/EUR and CZK/USD exchange rates expressed as monthly averages). All of these series have monthly values and were tracked in the following periods of time: CR exports, January 1996 – February 2011 (182 observations); CZK/USD exchange rate, January 1996 – April 2011 (184 observations); and CZK/EUR exchange rate, January 1999 – April 2011 (148 observations).

The analysis and all calculations were carried out using the SCA software.

Methodology

We applied the SARIMA models in the general form to the time series analysis.

$$\phi_p(B)\Theta_P(B^L)(1-B)^d(1-B)^D Y_t = \theta_q(B)\Theta_Q(B^L)\varepsilon_t$$

In addition to the usual means (ACF, PACF), other functions (EACF, IACF) and methods (SCAN, corner table) were used for model identification. All analyzed series were non-stationary and had to be transformed (by standard and seasonal differences) in order to

achieve stationarity. The stationarity was tested with the aid of several different criteria – unit root tests, homoscedasticity tests, etc.
We calculated the cross-correlation function (CCF) in order to establish linear dependence between the transformed (i.e., already stationary) series. This value confirmed the linear dependence between the Exports and CZK exchange rate series. After that, a model with transfer function in the general form was set up,

$$Y_t = c + \nu_0 X_t + \nu_1 X_{t-1} + \nu_2 X_{t-2} + \ldots + \nu_K X_{t-K} + \frac{1}{(1-\phi_1(B))(1-\Phi_1(B^L))}\varepsilon_t$$

in which the output series Y_t stands for the exports (after the respective transformations) and the input series X_t stands for the CZK exchange rate (also after the respective transformations), and the noise series is the last component of the model N_t. We used the LTF method for estimating the parameters – cf. e.g. [5] and [6]. The resulting model was used for calculating predictions.

Analysis of the CR exports time series – Model 1

First, we will find a suitable SARIMA model for export series. After a thorough analysis and studying ACF, PACF, EACF, IACF, unit root tests and homoscedasticity tests, we derived a rather complicated model (cf. the computer output):

$$(1-0.291B^3 - 0.249B^5 + 0.255B^{10})Y_t = (1-0.527B)(1-0.595B^{12})\varepsilon_t$$

where $Y_t = (1-B)(1-B^{12})\,Export_t$, ε_t is the white noise. This model has been successfully verified and proven as fully adequate; this fact is also indicated by the index of determination, which is equal to 0.978.

```
                                              1      12
EXPORTN    RANDOM     ORIGINAL     (1-B  ) (1-B  )
-----------------------------------------------------------------------
PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE

   1    TH1     EXPORTN    MA       1      1      NONE       .5274      .0708    7.45
   2    TH12    EXPORTN    MA       2     12      NONE       .5950      .0690    8.62
   3    PHI3    EXPORTN    AR       1      3      NONE       .2906      .0757    3.84
   4    PHI5    EXPORTN    AR       1      5      NONE       .2490      .0740    3.36
   5   PHI10    EXPORTN    AR       1     10      NONE      -.2551      .0750   -3.40

EFFECTIVE NUMBER OF OBSERVATIONS . .          159
R-SQUARE . . . . . . . . . . . . . .          .978
RESIDUAL STANDARD ERROR. . . . . . .   .831756E+04
```

Let us now set up a model for the time series of the CZK/USD exchange rate (hereinafter the USD series). The corresponding SARIMA for this series is:

$$(1-B)X_t = (1+0.295B)\varepsilon_t$$

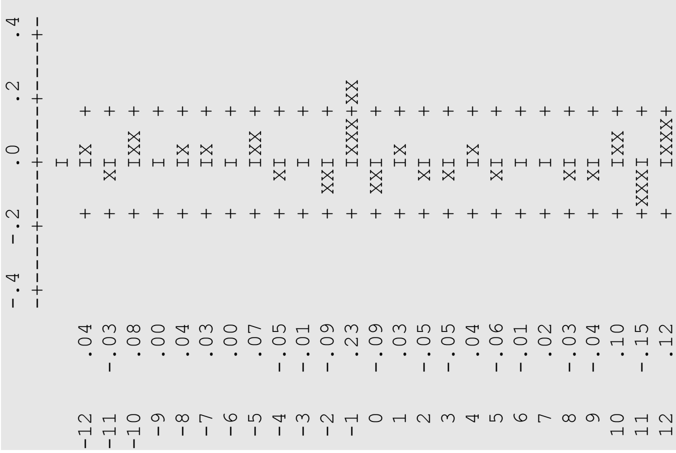
where X_t = USD_t and the index of determination is equal to 0.987. Both the model and the chart show that the USD series had to be differentiated – both standard and seasonal differentiation took place.

```
                                                1
  USDN     RANDOM     ORIGINAL     (1-B  )
 ----------------------------------------------------------------------
 PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
    LABEL      NAME    DENOM.                 TRAINT               ERROR  VALUE

  1   TH1      USDN      MA       1      1     NONE     -.2952     .0707  -4.17

 EFFECTIVE NUMBER OF OBSERVATIONS . .           181
 R-SQUARE . . . . . . . . . . . . . .          .987
 RESIDUAL STANDARD ERROR. . . . . . .   .807250E+00
```

On the SC… …orrelation function, now between stationa… …ion determines both the intensity and the… …the value of the cross-correlation functi…

```
 NAMES OF TH…                                                  N
 EFFECTIVE N…                                                  9
 STANDARD DE…                                                  0
 MEAN OF THE…                                                  8
 STANDARD DE…                                                  3
 T-VALUE OF …                                                  8

 CORRELATION…
 CROSS CORRE…

  1- 12                                                          .12
  ST.E.                                                          .08

 CROSS CORRE…

  1- 12                                                          .04
  ST.E.                                                          .08
```

CCF chart:



Both the calculated values and the chart of CCF (at the 95% confidence interval) imply that there is only one significant value of CCF, namely, at time *t-1*. This means there is a significant linear dependence between the (differentiated) Exports time series at time *t* and (again differentiated) USD times series at time *t-1*. Hence we can try to identify a transfer function model.

We utilize the LTF method [5] for such identification. The only significant weight which occurs is v_1; that is, the output (Exports) series depends on the input (USD) series' value with a time delay equal to 1. This was already indicated by the CCF chart. All other weights are insignificant. After laborious identification we get the following model for the error series:

```
                                              1      12
 EXPORT    RANDOM     ORIGINAL     (1-B  ) (1-B  )
                                              1      12
  USDN     RANDOM     ORIGINAL     (1-B  ) (1-B  )
---------------------------------------------------------------------
PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD      T
   LABEL       NAME    DENOM.                TRAINT               ERROR   VALUE

  1    V1      USDN    NUM.      1      1     NONE  1178.7950  487.7178    2.42
  2   TH1     EXPORT                                                        7.17
  3   TH12    EXPORT                                                        8.57
  4   PHI3    EXPORT                                                        4.05
  5   PHI5    EXPORT                                                        3.43
  6  PHI10    EXPORT                                                       -3.34

EFFECTIVE NUMBER OF
R-SQUARE . . . . .
RESIDUAL STANDARD E
```

with the resulting transfer

$(1-0.299B^3-0.25$ ... $.595B^{12})\varepsilon_t$

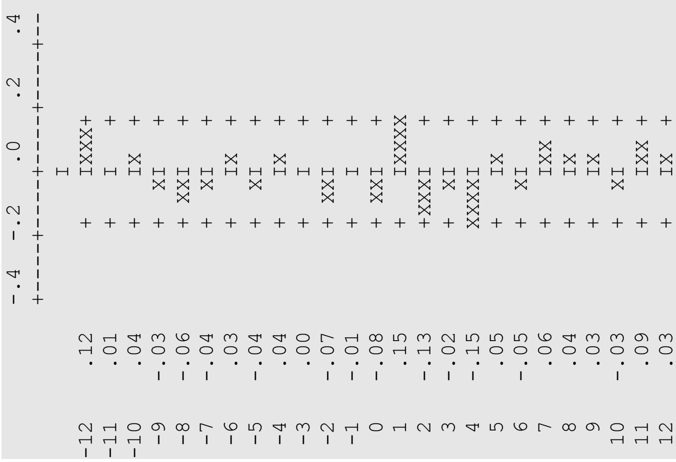
where

$Y_t =$ ...

This model has been succ... ity of the model is also indicated by the in...

The values of cross co... nput series and residuals of transfer fu... m the following outputs:

CCF charts:




```
CORRELATION        BETWEEN    RESY3     AND    RESY2 IS   -.08
CROSS CORRELATION BETWEEN    RESY2(T)  AND    RESY3(T-L)

  1- 12     .15 -.13 -.02 -.15  .05 -.05  .06  .04  .03 -.03  .09  .03
  ST.E.     .08  .08  .08  .08  .08  .08  .08  .08  .08  .08  .08  .08

CROSS CORRELATION BETWEEN    RESY3(T)  AND    RESY2(T-L)

  1- 12    -.01 -.07  .00  .04 -.04  .03 -.04 -.06 -.03  .04  .01  .12
  ST.E.     .08  .08  .08  .08  .08  .08  .08  .08  .08  .08  .08  .08
```

We can therefore observe that the Exports time series (after standard and seasonal differentiation) at time *t* depends on its past values (with the time shift equal to 3, 5 and 10), the values of the CZK/USD exchange rate time series (after standard and seasonal differentiation) at time *t-1*, and the past values of the error series (with the seasonal parameter).

Let us have a look at the predictions for which the model was sought in the first place. At the time of writing this paper (beginning of May 2011), the Exports time series values were only known until February 2011, but the CZK/USD exchange rate time series values were also known till April 2011 (CNB publishes the exchange rate values on its website at 16:00 hours every day). We are thus able to put the curremt exchange rate values into the model and estimate the CR exports for the next three period much more accurate way than on the basis of the input series values only. The reason is that we do not utilize estimates of future values of the input series.

Tab. 1: Predictions of the CR exports time series (Model 1)

	Predictions	
Month	SARIMA model	TFM
March	246,953 (8318)	245,283 (8171)
April	235,986 (9200)	234,640 (9162)

The tabled values show predictions for the original seasonal ARIMA model and for the transfer function model; the standard deviation values of the predictions are given in parentheses after the estimated value. We can see that the TFM predictions and SARIMA predictions are very similar. We can further observe that the SARIMA predictions for both months are higher than the TFM ones.

Analysis of the CR export time series – Model 2

Let us now identify a similar transfer function model, but the CZK/EUR exchange rate (EUR series) will be taken for the input series. The most suitable model is identified as

$$(1-B)X_t = (1+214B)\varepsilon_t$$

with the index of determination equal to 0.987, where X_t = Euro_t. We can clearly see from the model that standard differentiation of the EUR series was necessary to achieve stationarity.

```
  EURN     RANDOM     ORIGINAL     (1-B  )
----------------------------------------------------------------------
PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE

  1   THI1     EURN      MA       1      1     NONE     -.2141     .0818  -2.62

EFFECTIVE NUMBER OF OBSERVATIONS . .            145
R-SQUARE . . . . . . . . . . . . . .            .987
RESIDUAL STANDARD ERROR. . . . . . .     .434405E+00
```

We have to realize that we have data from a different period of time for the Exports time series, namely, years 1999 – 2011. We apply the previous model but with different estimates of its parameters:

$$(1-0.279B^3-0.291B^5+0.269B^{10}-0.166B^{12})Y_t = (1-0.538B)(1-0.749B^{12})\varepsilon_t$$

and the index of determination is equal to 0.968.

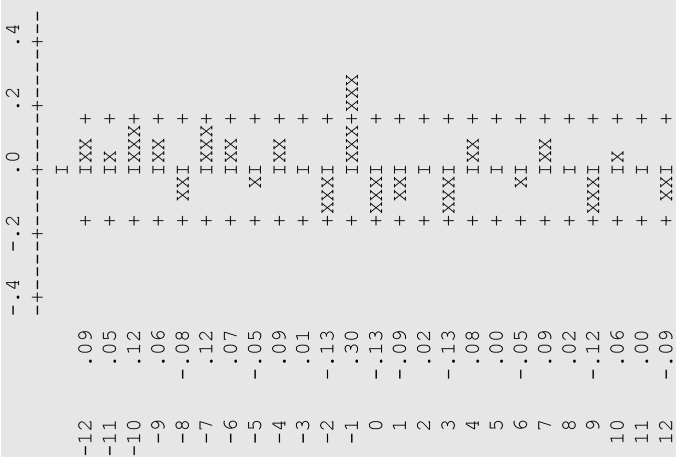
```
                                         1      12
EXPORTN    RANDOM     ORIGINAL     (1-B  ) (1-B  )
```

PARAMETER LABEL		VARIABLE NAME	NUM./ DENOM.	FACTOR	ORDER	CONS-TRAINT	VALUE	STD ERROR	T VALUE
1	TH1	EXPORTN	MA	1	1	NONE	.5381	.0824	6.53
2	TH12	EXPORTN	MA	2	12	NONE	.7485	.0931	8.04
3	PHI3	EXPORTN	AR	1	3	NONE	.2788	.0823	3.39
4	PHI5	EXPORTN	AR	1	5	NONE	.2912	.0806	3.62
5	PHI10	EXPORTN	AR	1	10	NONE	-.2687	.0821	-3.27
6	PHI12	EXPORTN	AR	1	12	NONE	.1657	.1041	1.59

```
EFFECTIVE NUMBER
R-SQUARE . . . .
RESIDUAL STANDAR
```

The cross-corre [...] n stationary series $(1-B)(1-B^{12})Y_t$ (den [...] $-B)X_t$ (denoted by EURND1). Let us ide [...] nce.

```
TIME PERIOD ANAL
NAMES OF THE SER
EFFECTIVE NUMBER
STANDARD DEVIATI
MEAN OF THE (DIF
STANDARD DEVIATI
T-VALUE OF MEAN

CORRELATION
CROSS CORRELATIO

  1- 12    -.09                                                      9
  ST.E.     .09                                                      9

CROSS CORRELATIO

  1- 12     .30                                                      9
  ST.E.     .09                                                      9
```



Both the calculated values and the chart of CCF (at the 95% confidence interval) imply that there is only one significant value of CCF, namely, at time t-1. This means there is a significant linear dependence between the (differentiated) Exports time series at time t and (again, differentiated) EUR time series at time t-1. Let us identify a transfer function model.

We again utilize the LTF method [5] for such identification. The only significant weights which occur are v_0 and v_1, that is, the output (Exports) series depends on the input (EUR) series value with a time delay equal to 1 (where both series have been differentiated as specified above). This was already indicated by the CCF chart. All other weights are insignificant. After laborious identification we get the following model for the error series:

```
                                              1      12
EXPORTN    RANDOM     ORIGINAL     (1-B  ) (1-B  )
                                              1
  EURN     RANDOM     ORIGINAL     (1-B  )
-----------------------------------------------------------------------
PARAMETER   VARIABLE  NUM./  FACTOR  ORDER   CONS-     VALUE      STD     T
   LABEL       NAME    DENOM.                TRAINT               ERROR  VALUE

  1    V0      EURN    NUM.      1      0     NONE -5328.3755 1607.9127  -3.31
  2    V1      EURN    NUM.      1      1     NONE  6420.6092 1633.0366   3.93
  3   TH1     EXPORTN   MA       1      1     NONE      .5344      .0821   6.51
  4   TH12    EXPORTN   MA       2     12     NONE      .7466      .0804   9.29
  5   PHI3    EXPORTN   AR       1      3     NONE      .3008      .0793   3.79
  6   PHI5    EXPORTN   AR       1      5     NONE      .2522      .0752   3.35
  7  PHI10    EXPORTN   AR       1     10     NONE     -.2284      .0767  -2.98
  8  PHI12    EXPORTN   AR       1     12     NONE      .1846      .0943   1.96

EFFECTIVE NUMBER OF OBSERVATIONS . .           121
R-SQUARE . . . . . . . . . . . . . .          .972
RESIDUAL STANDARD ERROR. . . . . . .    .819847E+04
```

The resulting transfe

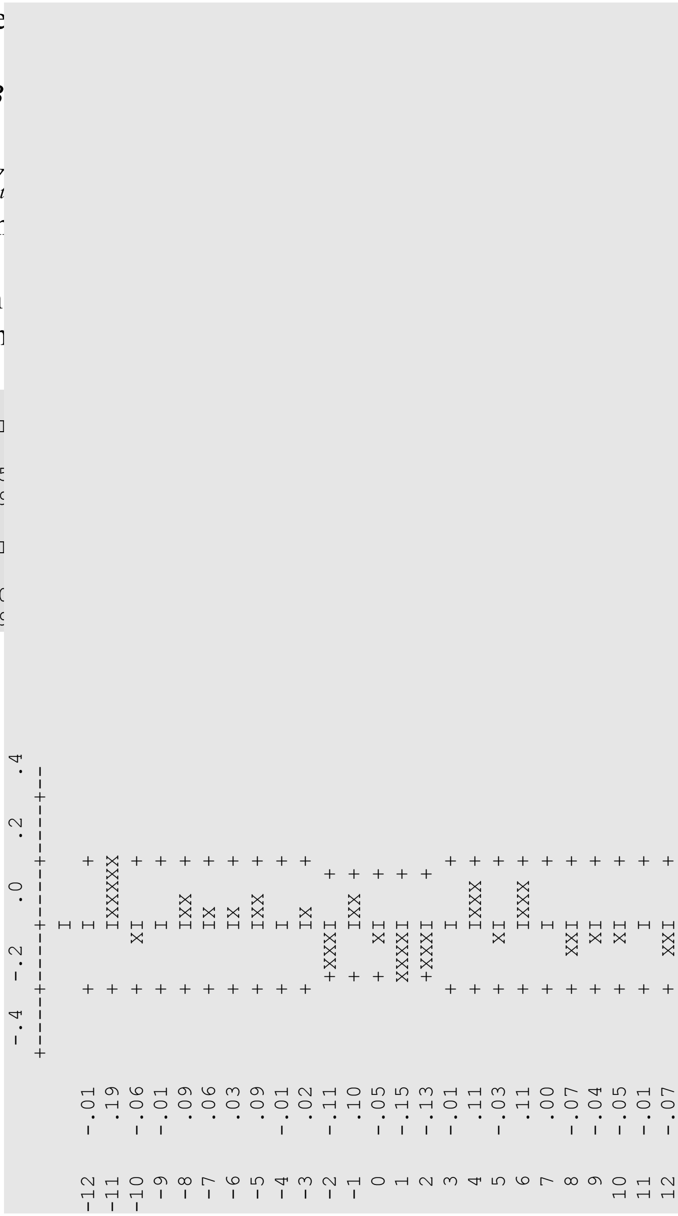
$(1-0.301B^3-0.252B$ … $.534B)(1-0.747B^{12})\varepsilon_t$

where

Y_t … o_t

This model has beer … Quality of the model is also indicated by … . The values of cross correlation function … residuals of transfer function model are n … :

```
CORRELATION
CROSS CORRELATI

  1- 12    -.15 …                                              .07
  ST.E.     .09 …                                              .10

CROSS CORRELATI

  1- 12     .10 …                                              .01
  ST.E.     .09 …                                              .10
```



We can therefore observe that the Exports time series (after standard and seasonal differentiation) at time *t* depends on its past values (with the time shift equal to 3, 5,10 and 12), the values of the CZK/EUR exchange rate time series (after standard differentiation) at time point *t-1*, and the value of the error series.

Let us compare the predictions obtained by the SARIMA and TFM models:

Tab. 2: Predictions of the CR exports time series (Model 2)

	Predictions	
Month	SARIMA model	TFM
March	246,713 (8749)	242,861 (8519)
April	235,422 (9638)	233,944 (9335)

The tabled values show predictions for the original seasonal ARIMA model and for the transfer function model; the standard deviation values of the predictions are given in parentheses after the estimated value. The SARIMA predictions for all three months are higher than the TFM predictions.

Now we can compare predictions within the SARIMA and TFM models for Model 1 and Model 2.

Tab. 3: Predictions of the CR exports – Model 1 vs. Model 2

	Model 1 (USD)		Model 2 (EUR)	
	SARIMA	TFM	SARIMA	TFM
March	246,953	245,283	246,713	242,861
April	235,986	234,640	235,422	233,944

Analysis of the CR import time series – Model 3

We repeat whole analysis for import series. We build TFM model with with output series Import and input series USD (EUR). The will work by the same way and we do not write the details of our analysis. Only final results are published without computer outputs.

The most suitable SARIMA model for Import series is

$$(1-0.241B^3-0.273B^5+0.292B^{10}-0.282B^{22})Y_t=(1-0.561B)(1-0.642B^{12})\varepsilon_t$$

where $Y_t=(1-B)(1-B^{12})\,Import_t$, ε_t is the white noise. This model has been successfully verified and proven as fully adequate; this fact is also indicated by the index of determination, which is equal to 0.971.

The corresponding SARIMA for this USD series we know from previous analysis:

$$(1-B)X_t=(1+0.295B)\varepsilon_t$$

where X_t = USD_t and the index of determination is equal to 0.987

The final TFM model is

$$(1-0.279B^3-0.233B^5+0.283B^{10}-0.253B^{22})Y_t=$$
$$=2102*X_{t\text{-}1}-1982*X_{t\text{-}2}+(1-0.539B)(1-0.64B^{12})\varepsilon_t$$

where

$$Y_t=(1-B)(1-B^{12})*Import_t \quad \text{and} \quad X_t=(1-B)*USD_t$$

This model has been successfully verified and proven as fully adequate. Quality of the model is also indicated by the index of determination, which is equal to 0.973.
The tabled values show predictions for the original seasonal ARIMA model and for the transfer function model

Tab. 4: Predictions of the CR imports time series (Model 3)

	Predictions	
Month	SARIMA model	TFM
March	227,050 (8421)	228,402 (8109)
April	216,590 (9197)	213,865 (9089)

Analysis of the CR import time series – Model 4

The most suitable model is known from previous analysis

$$(1-B)X_t = (1+214B)\varepsilon_t$$

with the index of determination equal to 0.987, where X_t = Euro$_t$. The most suitable SARIMA model for Import series is

$$(1-0.237B^3-0.248B^5+0.28B^{10}+0.286B^{22})Y_t = (1-0.546B)(1-0.622B^{12})\varepsilon_t$$

with index of determination 0.954.

The resulting transfer function model is

$$(1-0.235B^3-0.178B^5+0.217B^{10}+0.224B^{22})Y_t =$$
$$= -4216*X_t+7146*X_{t-1}-3734*X_{t-2}+(1-0.481B)(1-0.696B^{12})\varepsilon_t$$

where

$$Y_t = (1-B)(1-B^{12})*Import_t \quad \text{and} \quad X_t = (1-B)*Euro_t$$

Tab. 5: Predictions of the CR imports time series (Model 4)

	Predictions	
Month	SARIMA model	TFM
March	227,601 (9093)	231,341 (8846)
April	216,687 (9987)	217,492 (9959)

Now we can compare predictions within the SARIMA and TFM models for Model 3 and Model 4.

Tab. 6: Predictions of the CR exports – Model 3 vs. Model 4

	Model 3 (USD)		Model 4 (EUR)	
Month	SARIMA	TFM	SARIMA	TFM
March	227,050	228,402	227,601	231,341
April	216,590	213,865	216,687	217,492

Conclusions

We have successfully proven a mutual relationship between the time series of Czech Republic exports (imports), expressed in million CZK in fixed prices, and the time series of the CZK/USD (CZK/EUR) exchange rate. The analysis led us to creating a model based on a transfer function. It turned out that with the growing values of the CZK exchange rate (weakening Czech Crown), the exports are also growing. Since the values of the CZK exchange rate are known two to three months before the export (import) values, such models enable us to predict the export (import) time series immediately after the end of the respective month. In comparison, the SARIMA predictions are higher than the predictions within the TFM models. Standard deviations for TFM forecasts are mostly smaller than standard deviations for SARIMA forecasts. An advantage of the TFM models is the fact that actual, observed values are used in the input series rather than the usual estimates.

REFERENCES

[1] Abraham, B. – Ledolter, J.: Statistical Methods for Forecasting, John Wiley & Sons, New York, 1983.
[2] Anderson, O. D.: Time series analysis and forecasting – Box-Jenkins approach. London, Butterworth 1976.
[3] Box, G. E. P. – Jenkins, G. M. – Reinsel, G.C.: Time series analysis, forecasting and control. Third edition. Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1994.
Forecasting and Time Series Analysis using the SCA Statistical System. Volume 1. Scientific Associates Corp. Oak Brook, Illinois, USA, 1991.
[4] Granger, C. W. J., Newbold , P.: Forecasting Economic Time Series. Academic Press. New York, 1986.
[5] Pankratz, Alan: Forecasting with dynamic regression models. John Wiley & Sons inc., New York, 1991.
[6] The SCA Statistical System. Reference manual for general statistical analysis. Scientific Associates Corp. Oak Brook, Illinois, USA, 1991.
[7] Wei, W. W. S.: Time series analysis – Univariate and multivariate methods. Redwood City, California, Addison-Wesley Publishing Company 1990.

Abstract

The goal of this paper is to prove a mutual relationship between the time series of the Czech Republic exports (imports), expressed in million CZK in fixed prices, and the time series of the CZK/USD (or CZK/EUR) exchange rate. This relationship can be described using a transfer function model. Such models can be utilized in predicting the CR export (import) time series. There is a time-delayed mutual dependence between the export (import) time series and the CZK/USD (or CZK/EUR) exchange rate time series. Since the values of the CZK exchange rate are known two to three months before the export values, such models enable us to predict the export time series immediately after the end of the respective month. Moreover, the model has an additional advantage – the prediction is based on actual values of the input series (and not predicted values, as is the usual case).