Measuring Italian well-being by modified TOPSIS (*Tecnique for Order Preference by Similarity to Ideal Solution)*

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1. Problem definition and objectives

Well-being has always been an issue of concern among economists, researchers and policy makers. As Des Gasper (2004) points out, it is not an easily definable term and its meanings and conceptions remain ambiguous. Among the different conceptualizations that have been provided, perhaps the most useful is to regard well-being as an abstraction, to be used to refer to whatever is assessed in an evaluation of a person's life situation or 'being'.

From an operational point of view, well-being is commonly conceived as a multidimensional concept, and this entails the need of searching a broad consensus about its major dimensions.

This paper attempts to measure the well-being, assumed as a multidimensional concept, of the Italian regions, whose disparities in income levels, poverty, safety and so on, are well known and form issues of economic and political debates.

For the measurement of well-being TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) technique, one of Multiple Criteria Decision Making (MCDM) methods, will be used, as adequate tools in evaluating complex system. In MCDM methods *m* non dominated alternatives are characterized by *n* attributes or criteria. In the context of regional well-being measurement, the Italian regions provide the alternatives and elementary indicators represent criteria; non dominated means that there is no region which excels in all the well-being indicators considered. The solution of the MCDM problem can be assessed as a composite index[1], one for each dimension, able to rank the regions in term of their well-being.

TOPSIS method offers a well suited approach to rank regions with regard to achieved well-being. The modified TOPSIS used in this work, as further defined, is a methodological tool capable of handling conflicting situations between dimensions, (e.g. economy and environmental), relations between indicators and incomparable units (Antucheviciene et al, 2010).

The remainder of this paper is structured as follows. The second section provides the list of dimensions and some example of indicators. Section three is intended to focus on the main topics concerning the modified TOPSIS method and outlines the three fundamental issues of normalizing, weighting and using Mahalanobis distance in the development of the TOPSIS solution. Section four shows the case study, the results obtained and a brief discussion. The final section offers some concluding remarks.

2. The dimensions of well-being and related indicators

Dimension means an aspect of something to be described.[2] The chosen dimension of well-being are eleven: 1) demographics; 2) environmental; 3) social protection, health and public health; 4) economy; 5) education; 6) science, technology and innovation; 7) justice and safety; 8) culture; 9) social cohesion; 10) energy; 11) territory. Each dimension may have a different number of indicators able to characterize it. These indicators are weighted and aggregated to yield composite indices of the specific domain.

Because of space constraints, in this paper only some example of dimension and indicators are given, i.e marriage rates per 1000 inhabitants, ageing index and infant mortality rates per 1000 live births for the

[1]The term composite is now used with the same meaning of synthetic, but in literature a distinction is made between synthetic and composite index: the former represent an aggregation of homogeneous components, while in the former the components are heterogeneous.

2 Alkire (2002)

demographic dimension; regional GDP per inhabitant for the economy dimension, and so on. The number of indicators is ten for any dimension, the only exception being energy (nine indicators). By choosing an equal number of indicators the bias due to a different amount of indicators in the results of the weighting system has been removed. B and C refer to the nature of the indicator: benefit (B) if larger attribute values are preferred or cost (C) if smaller attribute values are preferred.

3. The modified TOPSIS method in the case study

The Multiple Criteria Decision Making framework is used in making decision in presence of multiple conflicting criteria. The focus is on ranking and selecting from a set of alternatives (inter alia, Huang and Yoon, 1981 and 1987, Zanakis at al, 1998, S. Opricovic, G.-H. Tzeng, 2004). Such problems are widespread in real life decision situations and so, these methods have an extremely wide range of applications in environmental, socio-economic, financial and management context (Diakoulaki et al., 1995; Deng et al., 2000; Kim, Park e Yoon, 1997; Zhou et al., 2006; Parkan e Wu, 1999; Isiklar e Buyukozkan, 2007; Jee e Kang, 2000; Wang T.-C. e Hsu J.-C., 2004; Tzeng G.-H. e Lin G, 2005, Antucheviciene,et al, 2010). MCDM problems involving a finite number of pre-specified alternatives, as the one treated in this paper, are referred to by the acronym MADM.

As often remarked, MADM methods are especially useful when conflicts exist between attribute, with the need to deal with a methodological tool capable of finding a best compromise solution from all feasible alternatives assessed on multiple attributes. Zeleny's compromise solution concept and TOPSIS (Tecnique for Order Preference by Similarity to Ideal Solution) method of compromise ranking ((Zeleny, 1982; Hwang e Yoon, 1981) are well know and widely used to deal with that circumstance.

The fundamental of TOPSIS is that in real life a "Pareto solution" doesn't exist; in the context of regional well-being assessment this means that it is never found that in a given region all indicators of a single dimension will be al their best level, so that a compromise solution will be necessary. This solution is a composite index of a dimension that measure the relative closeness of each region with respect to an ideal situation of well-being in that dimension.

The basic principle of TOPSIS method is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance from the antithesis of the ideal solution (often called negative ideal or anti-ideal solution). What is called the ideal solution is the set of all the best values achievable by each attribute (profit attributes the maximum, cost attributes the minimum); negative ideal solution is just the contrary, i.e. the set of all the worst attribute values achievable (profit attributes the minimum, cost attributes the maximum).

MADM methods are based on a performance or decision matrix obtained by the evaluation of all the alternatives in terms of each criterion or attribute.

The present well-being MADM problem (decision matrix) has 21 alternative (19 regions and 2 self-governig provinces) $R_1,.....,R_i,..,R_n$ (i=1,2..,21) and 10 attributes ($X_1,.....,X_j,..,X_m$) j=1,..10 for each dimension of well-being. So x_j^i (i=1,..n; j=1…,m) represents the value assigned to the i-th region when it is evaluated regarding the j-th indicator; a column X_j, denotes the performances values of all the n regions linked to the j-th indicator and a row R_i (i=1,2,..,n) the whole regional performance (concerning all the m indicators). Without loss of generality all the indicators values are assumed positive.

Almost all MADM methods, and among these TOPSIS, require: i) a normalization rule to eliminate the units of criterion values; ii) predetermined information on the relative importance of the attributes, which is usually given by a set of normalized weights. TOPSIS has been modified as regards normalization rule (a double normalization), and the statistical distance to take the correlation between indicators into account when computing statistical distances (Mahalanobis in place of Euclidean distance).

The TOPSIS procedure consists of the following seven steps.

(1) Computing the normalized decision matrix.

The first normalized value in the TOPSIS method is now calculated by a linear normalization

$$p_j^i = \frac{x_j^i}{\sum_{i=1}^{n} x_j^i} \quad \text{i=1,…,n; j=1,….,m} \qquad (1)$$

This rule has been chosen coherently with the method used in estimating weights, as will be shown hereinafter. It's worth noting that this normalization procedure doesn't eliminate variation as a base of differentiation of economic entities. Moreover the sum of the normalized values is one.

(2) Computing the weighting coefficients

Several methods can be used in estimating weights, (rif. bibl. Olson) but TOPSIS usually include a set of weighting factors achieved by using the entropy concept. (Shannon CE, Weaver W, 1947). Diakoulaki et al., 1995; Deng et al., 2000; Yeh, 2002, Zhou et al., 2006 refer to this approach as providing the objective weights in MCDM methods. It' well know that Shannon' entropy concept is related to the expected information content emitted by the (normalized) data.

Denoting as $e(\mathbf{P}_j)_{rel}$ the relative entropy measure (being ln (n) the maximum value) of j-th normalized indicator:

$$e(\mathbf{P}_j)_{rel} = -\frac{\sum_{i=1}^{n} p_j^i \log p_j^i}{\ln(n)} \qquad (2)$$

The weight is thus assigned by:

$$d(P_j) = 1 - e(P_j)_{rel}$$

The linear normalization achieved by sum of values is especially suggested when weights are computed by the procedure using the Shannon's entropy concept. The weight stems from the comparison between the value of each indicator and the situation that shows the maximum of uncertainty (or the minimum of information). This means that the weight shows how much the available information diverges from the limit situation of complete absence of information. In other words, the weight measures the informative content of the data.

If there are m criteria, the relative weights are (for each of the G dimensions):

$$w_j(d_j rel) = \frac{d(\mathbf{P}_j)}{\sum_{j \in g} d(\mathbf{P}_j)} \quad \text{with g=1,..,G and} \quad \sum_{j \in g} w_j(d_j rel) = 1 \qquad (3)$$

(3) Standardizing the decision matrix by the arithmetic mean and the standard deviation, obtaining the matrix Z:

$$Z_j = \left(z_j^1, z_j^2, \ldots, z_j^n\right)', Z^i = \left(z_1^i, z_2^i, \ldots, z_m^i\right), \qquad (4)$$

Thus not only the average value but also variation is made uniform. This eliminates variation as a base of differentiation of economic entities and it is useful to apply Mahalanobis distance (using indifferently variance and covariance matrix or correlation matrix).

(4) Computing the weighted matrix

Multiply the matrix **Z** by the vector of weights previously calculated to result the weighted matrix **V**:

$$\mathbf{V}_j = w_j(d_j rel) * \mathrm{Z}$$

(5) Defining the ideal and negative-ideal solution

Define two vectors $\mathbf{V}^+, \mathbf{V}^-$ that identify, the first, the ideal region, that is the set of all the best attribute values achievable with each indicator (profit indicators the maximum, cost indicators the minimum); and, the second, the negative ideal region, i.e. the set of all the worst indicator values achievable (profit indicators the minimum, cost indicators the maximum). The vector $\mathbf{V}^i$ (the set composed of really observed indicators values) will be compared with the vectors $\mathbf{V}^+$ and $\mathbf{V}^-$:

$$\mathbf{V}^i = \left(v_1^i, v_2^i, \ldots, v_m^i\right)'; \ \mathbf{V}^+ = \left(v_1^+, v_2^+, \ldots, v_m^+\right); \ \mathbf{V}^- = \left(v_1^-, v_2^-, \ldots, v_m^-\right); \quad \text{i=1,..,n} \qquad (5)$$

(6) Computing the separation measures, using the Mahalanobis distance.

$$D^{i+} = d\left(\mathbf{V}^i, \mathbf{V}^+\right) = \sqrt{\left(v_j^i - v_j^+\right)^T R^{-1}\left(v_j^i - v_j^+\right)} \; ; \qquad (6)$$

$$D^{i-} = d\left(\mathbf{V}^i, \mathbf{V}^-\right) = \sqrt{\left(v_j^i - v_j^-\right)^T R^{-1}\left(v_j^i - v_j^-\right)} \quad \text{i=1,2,..,n.} \qquad (7)$$

where R^{-1} is the inverse of the correlation matrix.

(7) Computing the relative closeness to the ideal solution.
Compute a relative measure ranking the Italian regions with regard to well-being that provide the solution of the present MADM problem:

$$IQ_g^i = \frac{\sqrt{\left(v_j^i - v_j^-\right)^T R^{-1}\left(v_j^i - v_j^-\right)}}{\sqrt{\left(v_j^i - v_j^-\right)^T R^{-1}\left(v_j^i - v_j^-\right)} + \sqrt{\left(v_j^i - v_j^+\right)^T R^{-1}\left(v_j^i - v_j^+\right)}} \qquad \text{i=1,2,..,n} \qquad (8)$$

The relative closeness of the region R_i with respect to V^+ is briefly defined as

$$IQ_g^i = \frac{D^{i-}}{D^{i-} + D^{i+}} \quad \text{i=1,2,..,n} \qquad (9)$$

The alternative that ranks one is the closest to the ideal solution and presents the separation from the negative ideal solution the most, according to the descending order of closeness coefficient.
It is worth noting that the ranking rule in equation doesn't consider the relative importance of the two distances D_i^+ and D_i^-, although the latter usually should be a major concern in real life decision making (Opricovic and Tzeng, op.cit).

4. Results and discussion
As pointed out above, the selected data for measuring regional well-being cover 10 indicators for the 19 different Italian regions and 2 self-governing provinces. The source of data is ISTAT (Italian National Institute of Statistics). Four points can be made regarding the data: 1) the selected indicators are those usually adopted in well-being analysis; 2) the assignment of indicators to one of the set of already defined dimensions is not unique, because the same indicator could be used in another dimension, depending on the judgement of the researcher; 3) the reference years are 2007-2008 relating to availability of data for the major number of indicators; 4) the classification of the indicators into benefit or cost class is sometime almost certainly matter of debate, because the same indicator could be intended as benefit or as a cost.
Ranks assigned to Italian regions with regard to each dimension of well-being are reported in Table 1 in descending order. Table 2 shows the ranks for each dimension and each region. Because of space constraints only results on six dimension are showed (environment; economy; science, technology and innovation; justice and safety; social cohesion; territory).
In Table 1 Italian regions that perform better are different for the selected dimension: Valle d'Aosta for environment dimension; Veneto for economy; Piemonte for science, innovation and technology; Trento for justice and safety; Lombardia for social cohesion; Bolzano for territory (local development).
Table 2 shows the ranks assigned to each region respect to each dimension: this table can provide some useful indication for national and local policies. The lecture of Table2 by row provides an indication for strong actions in one or more dimensions at local level. The lecture of Table2 by columns shows an indication for strong actions in each region at national level.

Table 1- The ranks assigned by IQ_g^i indices

D2 Environment		D4 Economy		D5 Science, technology and innovation		D6 Justice and safety		D8 Social cohesion		D10 Territory	
Region		Region		Region		Region		Region		Region	
Valle d'Aosta	1	Veneto	1	Piemonte	1	*Trento*	1	Lombardia	1	*Bolzano*	1
Trento	2	*Bolzano*	2	Emilia-Romagn	2	Basilicata	2	Veneto	2	*Trento*	2
Bolzano	3	*Trento*	3	Friuli-Venezia (	3	*Bolzano*	3	*Trento*	3	Valle d'Aosta	3
Toscana	4	Marche	4	Lombardia	4	Friuli-Venezia (	4	Toscana	4	Veneto	4
Umbria	5	Piemonte	5	Veneto	5	Valle d'Aosta	5	Emilia-Romagn	5	Toscana	5
Molise	6	Friuli-Venezia (	6	Liguria	6	Marche	6	Marche	6	Emilia-Romagn	6
Piemonte	7	Lombardia	7	Campania	7	Puglia	7	Lazio	7	Friuli-Venezia (	7
Friuli-Venezia (	8	Emilia-Romagn	8	Toscana	8	Molise	8	Umbria	8	Lombardia	8
Lombardia	9	Abruzzo	9	Abruzzo	9	Abruzzo	9	Piemonte	9	Marche	9
Veneto	10	Umbria	10	Marche	10	Sardegna	10	Friuli-Venezia (	10	Abruzzo	10
Emilia-Romagn	11	Toscana	11	Lazio	11	Toscana	11	Liguria	11	Piemonte	11
Basilicata	12	Valle d'Aosta	12	Umbria	12	Sicilia	12	*Bolzano*	12	Sardegna	12
Liguria	13	Liguria	13	*Bolzano*	13	Piemonte	13	Abruzzo	13	Liguria	13
Marche	14	Molise	14	*Trento*	14	Veneto	14	Puglia	14	Umbria	14
Lazio	15	Lazio	15	Sicilia	15	Lombardia	15	Valle d'Aosta	15	Lazio	15
Sardegna	16	Basilicata	16	Puglia	16	Umbria	16	Sardegna	16	Molise	16
Abruzzo	17	Puglia	17	Valle d'Aosta	17	Lazio	17	Campania	17	Sicilia	17
Calabria	18	Sardegna	18	Basilicata	18	Emilia-Romagn	18	Calabria	18	Basilicata	18
Campania	19	Campania	19	Calabria	19	Liguria	19	Basilicata	19	Campania	19
Sicilia	20	Calabria	20	Sardegna	20	Calabria	20	Molise	20	Puglia	20
Puglia	21	Sicilia	21	Molise	21	Campania	21	Sicilia	21	Calabria	21

Table 2 -The ranks for each dimension and each region

Regioni	D2 Environment	D4 Economy	D5 Science, technology and innovation	D6 Justice and safety	D8 Social cohesion	D10 Territory
Piemonte	7	5	1	13	9	11
Valle d'Aosta	1	12	17	5	15	3
Lombardia	9	7	4	15	1	8
Liguria	13	13	6	19	11	13
Bolzano	3	2	13	3	12	1
Trento	2	3	14	1	3	2
Veneto	10	1	5	14	2	4
Friuli-V.G.	8	6	3	4	10	7
Emilia-Rom.	11	8	2	18	5	6
Toscana	4	11	8	11	4	5
Umbria	5	10	12	16	8	14
Marche	14	4	10	6	6	9
Lazio	15	15	11	17	7	15
Abruzzo	17	9	9	9	13	10
Molise	6	14	21	8	20	16
Campania	19	19	7	21	17	19
Puglia	21	17	16	7	14	20
Basilicata	12	16	18	2	19	18
Calabria	18	20	19	20	18	21
Sicilia	20	21	15	12	21	17
Sardegna	16	18	20	10	16	12

5 Concluding remarks

Empirical data confirm that there are many disparities among Italian regions: the gaps exist in all considered dimensions of well-being. Best performances are located in Northern regions, worst in Southern areas. Interesting exceptions, however, are found, as Basilicata for Science, Technology and Innovation.
The ranking operation gives a preference ranking on the set of alternatives. In the present framework, it provides a priority list of the regions, showing which of them need urgent and strong actions in one or more dimensions.
Using TOPSIS method in regional well being assessment seems a feasible approach for at least three main reasons:

- → Ideal solution is provided by real best performance in each criteria. Representing a level of well-being effectively achieved it could be considered as a first benchmark or a target to be accomplished by regions which perform worst.
- → Distances between alternatives and ideal (anti-ideal) solutions are clearly evidenced for each indicator, showing for any region how much large should be the effort to achieve better performance.
- → The weighting system usually linked to TOPSIS (based on Shannon entropy concept) allows to update the weights and to modify the list of the indicators. In a world with endless wants and needs and with numerous conflicting interests, a process of evaluation involving multiple dimensions and related trade-offs, substitutability is not only inevitable, but also warranted.

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