A Study on a Dynamic Stochastic Process for Traffic Congestion

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Introduction

Traffic congestion is a serious socio-economic problem causing nation's resource waste such as fuel and time, and environmental expense due to increasing demand for traffic. To resolve traffic congestion, proper management based on a good knowledge of existing traffic network is the most effective and important way as a policy maker. Understanding the current traffic condition should precede in order to develope optimal road network for efficient traffic flow and minimal congestion.

Queueing process and Speed-Flow-Density diagram are very useful to investigate the stochastic phenomena of traffic and assess the network performance. Queueing models have been widely applied to explain congestion for interrupted and uninterrupted traffic flows (Heidemann D, 1994; Vandaele et al., 2000; van Woensel and Vandaele, 2006). Such traffic flow model through queueing process have allowed the non-linear relationship between speed, flow and density, and described the complex stochastic travel condition with quite accurate results (Woensel and Cruz, 2009). Also, queueing models, compared to regression-type models, are robust for ourliers or extreme values in traffic data by unexpected weather, sudden accident, or temporary road construction. However, despite of these advantages, their queueing models do not consider the fundamental changes of road condition in traffic jam. They assumed the traffic condition remains stable above maximum flow in a symmetric quadratic function. In real world, the traffic condition seems fundamentally change when the road network is highly congested, and a new approach is required to reflect such non-symmetric change of traffic phenomena.

Network performance as well as traffic congestion cost are good basis to operate and maintain the traffic network, and furthermore they can be usefully utilized to establish transportation policy such as facility validation and economic analysis. The fundamental purpose of above efforts on traffic flow models is to maximize roadway capacity and minimize travel time in network. In other words, it aims to maximize the network performance and efficiency. Network performance through queueing model or S-F-D diagram itself can be hard to interpret for information users. As a policy maker, indices to reflect network performance would be easily interpreted and directly accepted. Department of transportation in US have reported several morbility indices such as individual traveler congestion, the nation's congestion problem, travel needs served (Schrank, Lomax, 2007), and Japan also have examined the time loss indices due to traffic congestion at government level. However, these indices represents point-in-time status regardless of any systematic or stochastic rule, and thus they are insufficient to suggest policy direction for continuous management.

Moreover, quantifying the external congestion costs contributes to understand the efficiency of the network (Woensel and Cruz, 2009). Since the optimal use of transportation system can be achieved with proper paying of external cost on the system, i.e., the additional cost for each additional user, researches have investigated the traffic congestion in terms of marginal cost. William Vickrey (1955) proposed the basic idea of paying for the additional fares of subway system in peak-time and high-traffic-section, and later he developed his idea for road congestion. His proposal on efficient congestion cost provoked lots of researches on other applications such as road pricing, airport congestion (Vickrey, 1969; Dewees, 1979; De Borger et al., 1996; Carlin and Park, 1970). Woensel and Cruz (2009) applied queueing theory to traffic flows and estimated the congestion cost, taking into account the inherent stochasticity of traffic.

In this paper, we aim to suggest an alternative approach to queueing models and illustrate the optimization method by Newton-Raphson algorithm. With the optimized parameter estimates for network area on major highway, we evaluate network performance indices in terms of congestion and travel time, and investigate the network efficiency in view of congestion costs.

The remainder of this paper is structured as follows. The next section introduces a dynamic queueing model with structural change, with an empirical analysis of Korean expressway. The third and fourth section explains network performance indices and congestion cost based on the proposed queueing model, and illustrates them with applications, respectively. The Last section concludes the paper with final remarks.

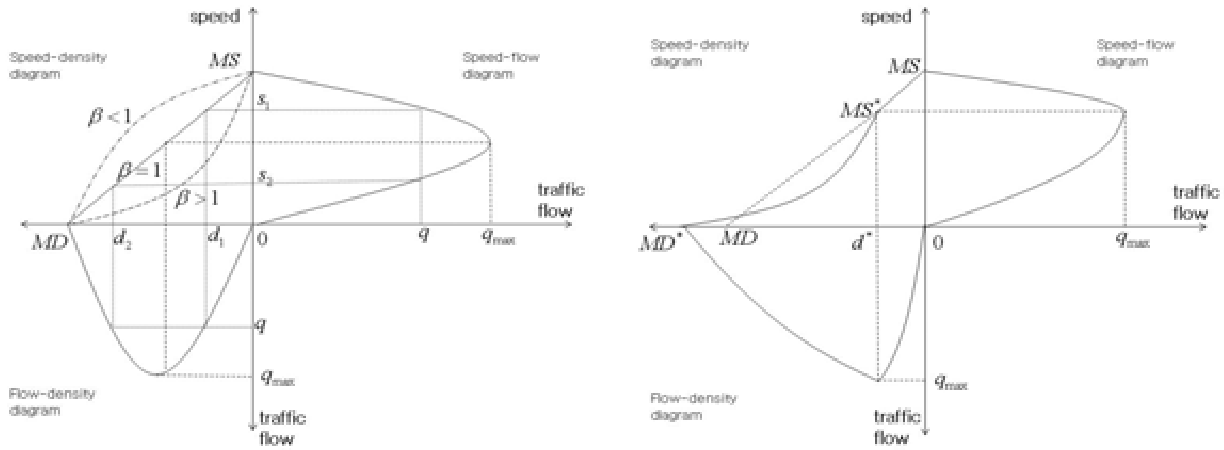
Dynamic Stochastic Process

M/G/1 queueing model assumes that inter-arrival time follow exponential distribution and service time follow general distribution. Inter-arrival rate λ is a product of density, (i.e., d), and maximum speed, (i.e., MS), and service time follows $G(1/\mu, \sigma^2)$. Therefore, speed of M/G/1 queue can be formed as $s = 2MS(MD - d)/\left[2MD + d(\beta^2 - 1)\right]$, where the c.v(coefficient of variation) of service time $\beta = \frac{\sigma}{1/\mu} = \sigma \times MS \times MD$. Vandaele et al. (2000) proposed relation of speed and flow based on M/G/1 queueing model as following. This also includes M/M/1 queueing model as a special case, when $\beta = 1$.

$$(1) \quad f(s,q) = 2MD \times s^2 + \left[q \times (\beta^2 - 1) - 2MD \times MS\right] \times s + 2MS \times q = 0.$$

Park and Jeon (2010) developed a dynamic M/G/1 model to reflect the change of traffic paradigm in congested network. When the road is highly congested above maximum flow, the density in the network increases while vehicle speed reduces. Although the nominal maximum speed is MS, practical maximum speed is limited to MS' less than MS above certain density d^*. Such fundamental change can be represented as a structural change of speed-density slope, or β, as seen in Figure 1. The relation between speed and flow β fundamentally changes from β_1 (for $d < d^*$) to β_2 (for $d \geq d^*$).

$$(2) \quad f(s,q) = \begin{cases} 2MD_1 s_1^2 + \left[q(\beta_1^2 - 1) - 2MS_1 MD_1\right] s_1 + 2qMS_1 = 0, & d < d^*, \\ [2MD_2 - (\beta_2^2 - 1)]s_2^2 + \left[q(\beta_2^2 - 1) - 2d^* - 2MS_2 MD_2\right] s + 2qMS_2 = 0, & d \geq d^*. \end{cases}$$



S-F-D diagram based on M/G/1 queueing model S-F-D diagram of Dynamic M/G/1 model

Figure 1: Speed-Flow-Density diagram

Table 1: *Theil* Statistics and Parameter Estimates for Each Region

Region	*Theil*		Parameter Estimates of Dynamic M/G/1 model						
	M/G/1	Dynamic M/G/1	MS_1	MD_1	β_1	d^*	MS_2	MD_2	β_2
Seoul-Singal	0.113	0.087	100	400	0.959	89	78.20	480	2.991
Singal-Seoul	0.118	0.032	110	400	0.971	90	85.78	480	3.246
Yangjae-Pangyo	0.169	0.044	100	240	0.222	60	85.11	300	2.603
pangyo-Yangjae	0.131	0.045	110	400	0.567	70	96.47	480	3.260

Traffic model generally assumes maximum speed and maximum density as known constant according to space limit (number of lanes) or technical limit (vehicle speed). However, the real observed maximum speed does not coincide with the ideal maximum speed. So does the maximum density. Moreover, the structural change point is not known in advance. Therefore, we deal them as variable parameters.

We use *Theil* statistics to find the optimized results for parameter estimates of queueing model, which is the best suitable model for current status of traffic network (Vandaele et al., 2000). *Theil* inequality coefficient (Theil, 1966) is useful to compare and evaluate the similarities of two different time series; 0 (accurate) to 1(inaccurate). It also can assess the accuracy of econometric forecasts compared to actual observation. Here, we find the optimal model with minimum *Theil* statistics through calibration of all possible value of parameter set $\{MS_1, MD_1, \beta_1, d^*, MS_2, MD_2, \beta_2\}$.

Calibration steps by nonlinear Newton-Raphson Algorithm are following.

Step1. Among all possible value of MS_i, change value by $5km/h$.

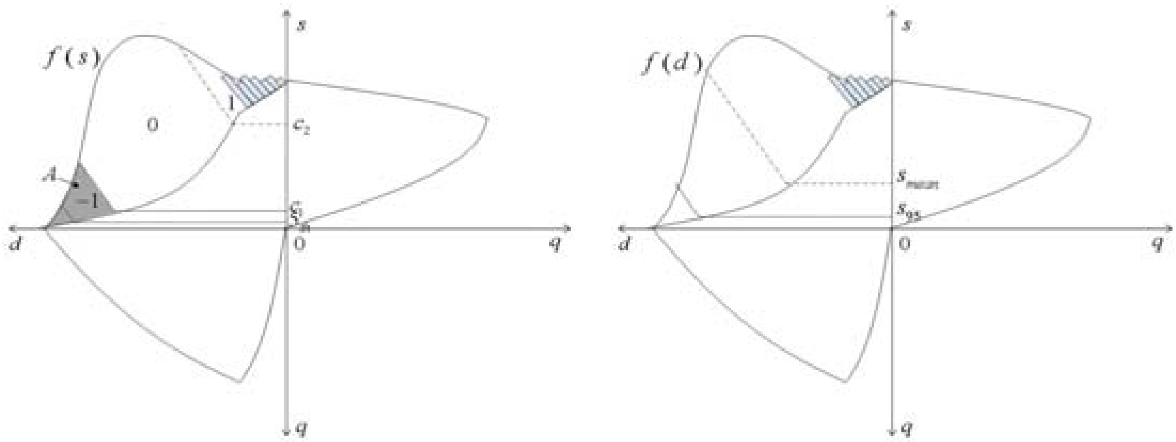
Step2. Among all possible value of MD_i, change value by $10veh/km$.

Step3. Among all possible value of d^*, change value by $1veh/km$.

Step4. Given values of MS_i, MD_i, d^*, find the optimal β_i minimizing the Objective function.

Network Performance Index

We develope a new index for network performance using the result of optimal queueing model



Traffic Congestion Index   Travel Reliability Index

Figure 2: Traffic Performance Indices using Empirical Distribution of Speed, Density

and its S-F-D diagram. We suggest dynamic indices via empirical distributions of speed and density, and illustrate them on S-F-D diagram.

Expected Flowing Index (EFI) is a weighted mean of empirical speed distribution to express expected flow, or traffic congestion. EFI near 1 implies free flow, while EFI near -1 does heavy traffic on the road, where weight in congested flow is -1, that of free flow is 1, and that of normal flow is 0. EFI using the distribution of M/G/1 speed is as following.

$$(3) \quad EFI = \int_0^{MS} I(s)f(s)ds.$$

where $I(s)$ is defined as follows.

$$I(s) = \begin{cases} -1, s \leq c_1, \\ 0, c_1 < s \leq c_2, \\ 1, s \geq c_2. \end{cases}$$

Buffer Index (BI) represents reliable expected travel time, or travel reliability. Travel time is a reverse of speed, and the ratio of travel time can be expressed as a reverse ratio of speed. BI is extra time added on freeflow travel time in order to arrive in time, and is increasing in congested traffic. It is a difference of PTI (planning time index) and TTI (travel time index), where TTI is an index of average travel time in normal condition, and PTI is a expecting time index to plan an important travel. The urban morbility report in US suggested TTI as a ratio of average travel time to freeflow travel time, and we convert it as a ratio of maximum speed to average speed. PTI is suggested as a ratio of 95th percentile travel time to freeflow travel time, and we convert it as a ratio of MS to 95th percentile speed.

$$(4) \quad BI = PTI - TTI = \frac{MS}{s_{95}} - \frac{MS}{s_{mean}}.$$

Table 2: Performance Index of Normal/Holiday for Each Region

Region	EFI		BI	
	Normal	Holiday	Normal	Holiday
Seoul-Singal	0.855	0.862	0.849	1.546
Singal-Seoul	0.903	0.952	1.162	0.206
Yangjae-Pangyo	0.976	0.942	0.129	0.117
Pangyo-Yangjae	0.831	0.971	2.873	0.058

Table 3: Total Cost for Each Region

Region	SU-SG	SG-SU	YJ-PG	PG-YJ
Maximum Flow(veh)	7003.52	7716.87	5105.44	6744.93
Total Cost(won)	19,857,715	19,331,764	12,866,289	19,429,079

Traffic Congestion Cost

Total congestion cost (TC), or cost of travel delay, can be seen as a function of value of time, flow and speed (De Borger et al., 1996; Woensel and Cruz, 2009).

$$(5)\quad TC_q = q \times C_q = q \times \frac{VOT}{s_q}.$$

Optimal use of a transportation facility cannot be achieved unless each additional user pays for the additional costs that this user imposes on all other users and on the facility itself. Marginal congestion cost (MC) is a proper concept for the network efficiency, because it is the additional social expense to maintain and operate the transportation system (Woensel and Cruz, 2009). The marginal congestion cost is defined as the extra cost due to a structural exogenous increase of traffic demand by 1 vehicle.

$$(6)\quad MC_q = \frac{\partial TC_q}{\partial q} = C_q + q \times \frac{\partial C_q}{\partial_q}.$$

It can be divided into two parts; internal cost (IC) and external cost (EC). IC is cost per vehicle in existing network condition, and is determined by speed of optimal queueing model.

$$(7)\quad IC_q = C_q = \frac{VOT}{s_q}.$$

EC is cost of additional vehicle, and can be expressed as follows.

$$(8)\quad EC_q = \begin{cases} \frac{(\beta_1^2-1)}{2MS_1(MD_1-q/s_1)s_1}\left[\frac{1+2MD_1/(\beta_1^2-1)+q/s_1}{(MD_1-q/s_1)}\right] VOT \times q & , d < d^*, \\ \frac{(\beta_2^2-1)}{2MS_2(MD_2-q/s_2)s_2}\left[\frac{1+2(MD_2-d^*)/(\beta_2^2-1)+(q/s_2-d^*)}{(MD_2-q/s_2)}\right] VOT \times q, & d \geq d^*. \end{cases}$$

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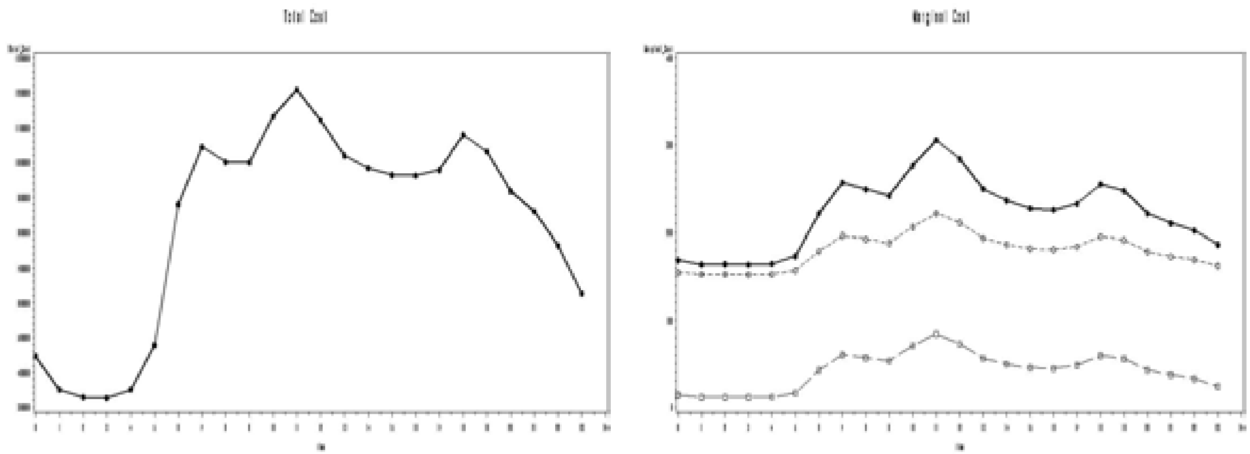

TC for SU-SG MC, IC and EC for SU-SG

Figure 3: Total, Marginal, Internal and External Congestion Cost

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