Bayesian Variable Sampling Plans for the Exponential Distribution with Progressive Hybrid Censoring

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In the past two decades, following the work of Lam (1990), there has been a growing literature on the development of Bayesian variable sampling plans for the exponential distribution based on different types of data including Type-I, Type-II, and random censoring; see Lin *et al.* (2008a) and the references contained therein. Recently, Chen *et al.* (2004) and Lin *et al.* (2008b, 2010) discussed the Bayesian variable sampling plans for exponential lifetime distribution under mixed or ordinary hybrid censoring, and progressive hybrid censoring schemes.

In the Type-I case, there are two ways to approach the optimal Bayesian plan, one is to condition on at least one failure in order for the MLE to exist (see Lin *et al.* 2008a, 2008b, 2010), and the other one is not to condition on that event and use an ad hoc estimate of the parameter when no failure occurs since the MLE does not exist in this case (see Lam 1994). Using the former approach and simulated optimization algorithm, Lin *et al.* (2010) pointed out that the minimum Bayes risks (MBR) under Type-I censoring, Type-I hybrid censoring, or Type-I progressive hybrid censoring are the same when the time-consuming cost and salvage are not included in the loss function. It is important to note that in over 56% of the selected cases in Table 1 of Lin *et al.* (2010), the values of MBR based on the former approach are always smaller than those based on the Lam's approach (with relative efficiencies in the range of 99.3–100%); but, the former approach is not uniformly better in that the efficiencies in some cases is slightly above 100% (with the highest value achieved being 100.2%).

Comparatively, employing the same techniques with the exact distribution of the MLE, the analogous conclusion does not occur in the Type-II case in that the progressive hybrid censoring plans are generally more efficient followed by ordinary hybrid censoring plans and then plans of Lam (1990) in terms of efficiencies. Thus, it is of great interest to use the same procedure to investigate the optimal sampling plans from an exponential distribution under both types of the adaptive progressive hybrid censoring schemes (APHCS) when a general loss function given in Eq. (1) below, which includes the sampling cost, the time-consuming cost, and the salvage, is used, and also to compare their performance with those of progressive hybrid censoring scheme (PHCS). An overview of these progressive hybrid censoring schemes and the related inferential methods can be found in Huang (2010).

Bayes Risk

Suppose that a lot of N items are presented for acceptance sampling and a sample of size n is taken from the lot. Given λ, the probability density functions of the maximum likelihood estimator (MLE) of the average lifetime $\theta = 1/\lambda$ from an exponential distribution with pdf $f(x) = \lambda e^{-\lambda x}$ for $x > 0$ and $\lambda > 0$ under (Type-I and Type-II) PHCS and APHCS (which are denoted by $f_{\hat{\theta}}(x)$) are all linear combination of gamma distributions (see Lin *et al.*, 2010, Huang, 2010). The larger θ, the larger expected lifetime. Thus, it is reasonable to reject a batch if $\hat{\theta}$ is small. It then leads to the following one-sided decision function:

$$\delta(\boldsymbol{X}) = \begin{cases} 1, & \hat{\theta} \geq \xi, \\ 0, & \text{otherwise}, \end{cases}$$

where $\boldsymbol{X}$ is the resulting failure times and $\delta(\boldsymbol{X}) = 1$ and 0 represent the decisions of accepting and rejecting the batch, respectively.

Let C_1 be the cost for inspecting an item, C_2 be the salvage incurred by an unfailed item in the inspection, C_3 be the cost per unit time used for life test, C_4 be the loss of rejecting the batch, $C_5\phi(\lambda)$ be the loss of accepting the batch, where $C_5 = (1-n/N)$. Assuming that $C_1, \ldots, C_5$ are non-negative, $C_1 > C_2 \geq 0$, and $\phi(\lambda) = a_0 + a_1\lambda + \cdots + a_k\lambda^k$ is a positive and non-decreasing function of λ for $\lambda \geq 0$. Noting that the loss of accepting or rejecting the batch without sampling is often greater than one can afford in many applications, therefore, our study will not include these two extreme cases. Combining all the losses and salvage, the loss function for the sampling plan $S(n, m, (R_1, \ldots, R_m), T, \xi)$, is usually defined as:

$$l(\delta(\boldsymbol{X}), \lambda) = C_1 n - C_2(n - m^*) + C_3\tau + (1 - \delta(\boldsymbol{X}))C_4 + \delta(\boldsymbol{X})C_5\phi(\lambda), \tag{1}$$

where

$$(m^*, \tau) = \begin{cases} (D, T) & \text{for Type-I APHCS}, \\ (m, X_{m:m:n}) & \text{for Type-II APHCS}, \\ (\min\{m, D\}, \min\{X_{m:m:n}, T\}) & \text{for Type-I PHCS}, \\ (\max\{m, D\}, \max\{X_{m:m:n}, T\}) & \text{for Type-II PHCS}. \end{cases}$$

Then, by assuming that the scale parameter λ has a conjugate gamma prior with density function

$$h(\lambda; a, b) = \frac{b^a}{\Gamma(a)}\lambda^{a-1}e^{-\lambda b},\ \lambda > 0, \tag{2}$$

and $\phi(\lambda)$ is of order $k = 2$ for the purpose of illustration, the Bayes risk for $\delta(\boldsymbol{X})$ is given by

$$\begin{aligned} R(n, m, R_1, \cdots, R_m, T, \xi) &= E[l(\delta(X), \lambda)] = E_\lambda\{E_{\boldsymbol{X}|\lambda}[l(\delta(\boldsymbol{X}), \lambda, n)|\lambda]\} \\ &= (C_1 - C_2)n + C_2 E_\lambda E_{\boldsymbol{X}|\lambda}[m^*|\lambda] + C_5(a_0 + a_1\mu_1 + a_2\mu_2) \\ &\quad + C_3 E_\lambda E_{\boldsymbol{X}|\lambda}[\tau|\lambda] + \sum_{\ell=0}^{2} C_\ell^* \frac{b^a}{\Gamma(a)} \int_0^\infty \int_0^\xi \lambda^{a+\ell-1}e^{-\lambda b} f_{\hat{\theta}}(x)\,dx\,d\lambda, \end{aligned}$$

where μ_1 and μ_2 are the first and second moments of λ about 0 and

$$C_\ell^* = \begin{cases} C_4 - C_5 a_0, & \ell = 0, \\ -C_5 a_\ell, & \text{otherwise}. \end{cases}$$

According to the selected progressive hybrid censoring scheme, $E_\lambda E_{\boldsymbol{X}|\lambda}[m^*|\lambda]$, $E_\lambda E_{\boldsymbol{X}|\lambda}[\tau|\lambda]$, and $\int_0^\infty \int_0^\xi \lambda^{a+\ell-1}e^{-\lambda b} f_{\hat{\theta}}(x)\,dx\,d\lambda$ can be easily determined. For instance, under Type-II APHCS, we can follow from the Eqs. (5) and (7) of Ng *et al.* (2009) and the identity

$$E_\lambda(\lambda^\vartheta e^{-\lambda\rho T}) = \frac{b^a}{\Gamma(a)} \int_0^\infty \lambda^{a+\vartheta-1} e^{-\lambda(b+\rho T)}\,d\lambda = \frac{b^a}{\Gamma(a)} \frac{\Gamma(a+\vartheta)}{(b+\rho T)^{a+\vartheta}}$$

to obtain the expression of $E_\lambda E_{\boldsymbol{X}|\lambda}[X_{m:m:n}|\lambda]$ for $a > 1$, and then from the Lemma 1 of Lin *et al.* (2010) with $f_{\hat{\theta}}(x)$ in Eq. (2.10) of Huang (2010) to have

$$\int_0^\infty \int_0^\xi \lambda^{a+\ell-1}e^{-\lambda b} f_{\hat{\theta}}(x)\,dx\,d\lambda = \sum_{d=0}^{m}\sum_{k=0}^{d} \frac{c_m \cdot c_{k,d}(R_1+1, \ldots, R_d+1)}{\prod_{k=1}^{m-d}(k + \sum_{i=d+1}^{m} R_i)} A_{\gamma_{d-k+1}, m, 0},$$

where $c_m = \prod_{i=1}^{m} \gamma_i$ with $\gamma_i = \sum_{k=i}^{m}(R_k+1)$, $c_{i,r}(\alpha_1,\ldots,\alpha_r) = \frac{(-1)^i}{\left\{\prod_{j=1}^{i}\sum_{k=r-i+1}^{r-i+j}\alpha_k\right\}\left\{\prod_{j=1}^{r-i}\sum_{k=j}^{r-i}\alpha_k\right\}}$, and $A_{\zeta,\eta,\kappa} = \frac{\Gamma(a+\ell)}{(b+\kappa T+\zeta T)^{a+\ell}} I_{S_{\zeta,\eta,\kappa}}(\eta, a+\ell)$. Here $S_{\zeta,\eta,\kappa} = \frac{\eta\xi-\zeta T}{b+\kappa T+\eta\xi}$ and $I_x(\alpha,\beta) = \frac{B_x(\alpha,\beta)}{B(\alpha,\beta)}$ is the distribution function of beta (α,β) distribution with $B_x(\alpha,\beta) = \int_0^x t^{\alpha-1}(1-t)^{\beta-1}dt$ being the incomplete beta function for $0 \le x \le 1$. Combining these two results as well as $E_\lambda E_{\boldsymbol{X}|\lambda}[m^*] = m$, we thus have the explicit expression for the Bayes risk of a sampling plan $S(n, m, (R_1, \ldots, R_m), T, \xi)$ under Type-II APHCS. Analogously, the explicit expressions for the Bayes risks of the sampling plans under other three progressive hybrid censoring schemes can be derived; see Huang (2010) for the detailed derivations.

Algorithm and Numerical Results

Denote the set of feasible values of $(n, m, (R_1, \ldots, R_m), T, \xi)$ by G. The optimal sampling plan $S(n^o, m^o, (R_1^o, \ldots, R_m^o), T^o, \xi^o)$ is the one that minimizes the Bayes risk $R(n, m, (R_1, \ldots, R_m), T, \xi) = E[l(\delta(\boldsymbol{X}), \lambda)]$ for $(n, m, (R_1, \ldots, R_m), T, \xi) \in G$. Thus, the steps for finding an optimal sampling plan in a class of possible sampling plans are as follows:

(a) set $n^* = n = 1$ and $m^* = m = 1$. Find optimal T and ξ, say T^* and ξ^*, to minimize $R(1, 1, (0), T, \xi)$. Set $R_S \equiv R(1, 1, (0), T^*, \xi^*)$.

(b) If n^* violates the condition given in Eq. (3) below, go to step **d**.

(c) Set $n = n + 1$. For $m = 1, \ldots, n$ and all possible choices of $(R_1, \ldots, R_m)$, find optimal T and ξ, say T' and ξ', to minimize $R(n, m, (R_1, \ldots, R_m), T, \xi)$. If $R(n, m, (R_1, \ldots, R_m), T', \xi') < R_S$, set $n^* = n$, $m^* = m$, $T^* = T'$, $\xi^* = \xi'$, $(R_1^*, \ldots, R_m^*) = (R_1, \ldots, R_m)$, and $R_S = R(n, m, (R_1, \ldots, R_m), T', \xi')$. Go to step **b**.

(d) $S(n^o, m^o, R_1^o, \ldots, R_m^o, T^o, \xi^o)$ is the optimal sampling plan with $n^* = n$, $m^* = m$, $T^* = T'$, $\xi^* = \xi'$, and $(R_1^*, \ldots, R_m^*) = (R_1, \ldots, R_m)$.

This algorithm is finite, that is, we can find an optimal sampling plan in a finite number of steps in the search. It is true in view of the facts that, for $n \ge 1$, $R(n^o, m^o, (R_1^o, \ldots, R_m^o), T^o, \xi^o) \le R(n, m, (R_1, \ldots, R_m), T_n, \xi_n)$, where $R(n, m, (R_1, \ldots, R_m), T_n, \xi_n) = \min_{T,\xi}\{R(n, m, (R_1, \ldots, R_m), T, \xi)\}$, and, from the definition of the loss function in Eq. (1), $R(n^o, m^o, (R_1^o, \ldots, R_m^o), T^o, \xi^o) \ge n^o(C_1 - C_2)$. Thus, the optimal sample size n^o satisfies the condition: for $n \ge 1$,

$$n^o \le \frac{R(n, m, (R_1, \ldots, R_m), T_n, \xi_n)}{C_1 - C_2}. \tag{3}$$

Since the expression of $R(n, m, (R_1, \ldots, R_m), T, \xi)$ is very complicated, the regular numerical optimizations such as Newton-Gauss and steepest descent methods are not applicable; hence, a simulated annealing algorithm (see Lin *et al.* 2010) is employed for the determination of an optimal sampling plan in our numerical examples below.

By setting $a = 1.5$, $b = 2.0$, $a_0 = 1.0$, $a_1 = 1.0$, $a_2 = 1.0$, $N = 1000$, $C_1 = 1.0$, $C_2 = 0.5$, $C_3 = 2.0$ and $C_4 = 5.0$ as the true values of the parameters and coefficients in the model (which we refer as **the original setting**), the optimal sampling plan is $S(4, 1, (3), 18.0196, 0.017)$ under Type-II APHCS with Bayes risk being 7.1844; and is $S(1, 1, (0), 21.3921, 0.0170)$ under Type-II PHCS with Bayes risk being 7.1011. Thus, the relative efficiency of the plan under Type-II PHCS to the plan under Type-II APHCS (**Eff2**) can be computed as $7.1011/7.1844 = 98.8\%$, which indicates that the difference between these two schemes is negligible.

In many situations, the parameters and coefficients are not known in advance. They may be estimated or be assigned subjectively by experimenters. Thus, there is a need to investigate how the change in efficiency for each censoring scheme when one of the selected parameters or coefficients used in the model has been misspecified. To demonstrate the analysis, we conduct a sensitivity analysis study with parameters a and b, and coefficients a_2, C_3, and C_4, respectively. The results are presented in Table 1. The efficiency reported as **Eff1** is the ratio of the MBR, $R(n^o, m^o, (R_1^o, \ldots, R_m^o), T^o, \xi^o)$, under the original setting and the one under the setting with one misspecified parameter or coefficient. It is easily seen that, the values of **Eff1** in over 63% (19/30) cases under Type-II APHCS are in the range of 85–115%; but, this does not occur consistently in that the efficiencies in some cases are below 32% or above 190% if one of the parameters or coefficients are chosen incorrectly. In contrast, there are about 37% (11/30) of the selected cases under Type-II PHCS that the efficiencies are in the range of 85–115% (with the lowest and the highest values achieved being 36.8% and 497.6%). It shows that the proposed optimal sampling plans under Type-II APHCS are generally much more robust than those under Type-II PHCS with regard to changes in the parameters and coefficients used in the model.

It is also important to compare the proposed sampling plans based on these two schemes. As expected, the values of MBR under Type-II PHCS except the case $C_3 = 0$ are all smaller than those under Type-II APHCS. However, among the 11 robust cases, the values of **Eff2** are in the range of 82.2–99.6%, which suggests that there is approximately no difference in efficiency between these two schemes.

Given that $D \geq 1$, similar results in terms of robustness and efficiency can also be observed in the Type-I case, as seen in Table 2. In general, the optimal sampling plans under Type-I APHCS are more robust than those under Type-I PHCS; and, from the settings of these two schemes, the relative efficiencies of the plans under Type-I PHCS to those under Type-I APHCS are often significantly larger than 115%. On the whole, we can conclude that for exponential distribution, the plans under APHCS, especially in the Type-I case, are generally more robust and efficient than those based on PHCS when a general loss function is used.

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Table 1: The MBRs and optimal sampling plans for some selected values of a, b, a_2, C_3 and C_4 under APHCS and PHCS in the Type-II case

a	b	a_2	C_3	C_4	Scheme	n^o	m^o	$(R_1^o, \cdots, R_m^o)$	T^o	ξ^o	MBR	Eff1(%)	Eff2(%)
1.5	2.0	1.0	2.0	5.0	APHCS	4	1	(3)	18.0196	0.0170	7.1844		
					PHCS	1	1	(0)	21.3921	0.0170	7.1011		98.8
2.0					APHCS	3	1	(2)	4.4317	0.1506	6.7917	105.8	
					PHCS	1	1	(0)	10.6491	0.1506	5.6288	126.2	
2.5					APHCS	2	1	(1)	3.5540	0.4657	6.9126	103.9	
					PHCS	1	1	(0)	6.7469	0.4676	5.7103	124.4	
3.0					APHCS	2	1	(1)	0.7569	0.7870	6.9878	102.8	
					PHCS	3	2	(0,1)	0.0169	0.7131	3.4059	208.5	
3.5					APHCS	2	1	(1)	0.2151	1.1080	7.0326	102.2	
					PHCS	2	1	(1)	0.0145	1.1080	5.3031	133.9	
4.0					APHCS	2	1	(1)	1.7355	1.4288	7.0347	102.1	
					PHCS	2	1	(1)	0.0127	1.4286	4.2075	168.8	
5.0					APHCS	2	1	(1)	0.0674	2.0698	6.9722	103.0	
					PHCS	2	1	(1)	0.0102	2.0700	1.4270	497.6	
1.5	0.8				APHCS	3	1	(2)	7.0134	1.0200	7.4685	96.2	
					PHCS	2	1	(1)	0.0136	1.0215	3.7891	187.4	
	1.0				APHCS	3	1	(2)	6.1897	0.8201	7.4974	95.8	
					PHCS	2	1	(1)	0.0170	0.8214	5.9307	119.7	
	3.0				APHCS	3	1	(2)	17.3696	0.0255	7.9389	90.5	
					PHCS	3	1	(2)	0.2330	0.0355	7.8989	89.9	99.5
	4.0				APHCS	3	1	(2)	6.4334	0.0340	8.9738	80.1	
					PHCS	3	1	(2)	0.3698	0.0340	8.9125	79.7	
	5.0				APHCS	2	1	(1)	50.1664	0.0426	12.9870	55.3	
					PHCS	1	1	(0)	53.4804	0.0426	11.0105	64.5	
	10.0				APHCS	2	1	(1)	14.3119	0.0851	22.7314	31.6	
					PHCS	1	1	(0)	106.9607	0.0851	19.2767	36.8	
	2.0	2.0			APHCS	4	1	(3)	8.0285	0.4204	7.9139	90.8	
					PHCS	4	3	(2*0,1)	0.0340	0.7043	6.5036	109.2	82.2
		2.5			APHCS	4	1	(3)	18.0813	0.6642	8.1161	88.5	
					PHCS	3	1	(2)	0.0340	0.6661	7.7033	92.2	94.9
		3.0			APHCS	4	1	(3)	15.4680	0.8851	8.2648	86.9	
					PHCS	3	1	(2)	0.0340	0.8870	7.3637	96.4	89.1
		5.0			APHCS	4	1	(3)	0.7582	1.6245	8.6104	83.4	
					PHCS	3	2	(0,1)	0.0340	1.4430	2.4687	287.6	
		7.5			APHCS	4	1	(3)	9.0591	2.3632	8.8203	81.5	
					PHCS	2	1	(1)	0.0340	2.3691	6.0193	118.0	
		10.0			APHCS	4	1	(3)	8.4516	2.9867	8.9406	80.4	
					PHCS	2	1	(1)	0.0340	2.9934	4.1788	169.9	
		1.0	0.0		APHCS	1	1	(0)	15.0155	0.0170	3.6923	194.6	
					PHCS	1	1	(0)	0.4800	0.0170	3.6923	192.3	
			1.0		APHCS	3	1	(2)	6.9000	0.0170	6.0203	119.3	
					PHCS	5	4	(3*0,1)	0.0340	0.4352	3.9550	179.5	
			3.0		APHCS	5	1	(4)	4.0155	0.0170	8.0817	88.9	
					PHCS	5	1	(4)	0.0771	0.2443	8.0489	88.2	99.6
			4.0		APHCS	5	1	(4)	18.5816	0.0170	8.8817	80.9	
					PHCS	5	1	(4)	0.0850	0.2801	8.8437	80.3	
			5.0		APHCS	5	1	(4)	14.6429	0.0170	9.6817	74.2	
					PHCS	5	1	(4)	0.0857	0.2407	9.6375	73.7	
			10.0		APHCS	5	1	(4)	16.9107	0.0170	13.6817	52.5	
					PHCS	5	1	(4)	0.1153	0.2292	13.6034	52.2	
			2.0	10.0	APHCS	4	1	(3)	11.3484	0.0170	7.2475	99.1	
					PHCS	1	1	(0)	21.3921	0.0170	7.1643	99.1	98.9
				15.0	APHCS	4	1	(3)	4.5194	0.0170	7.3107	98.3	
					PHCS	4	1	(3)	0.0340	0.0170	7.1770	98.9	98.2
				20.0	APHCS	4	1	(3)	5.9694	0.0170	7.3739	97.4	
					PHCS	4	1	(3)	0.0340	0.0170	7.1771	98.9	97.3
				30.0	APHCS	4	1	(3)	1.0250	0.0170	7.5002	95.8	
					PHCS	4	1	(3)	0.0340	0.0170	7.1771	98.9	95.7
				50.0	APHCS	4	1	(3)	15.3002	0.0170	7.7528	92.7	
					PHCS	4	1	(3)	0.0353	0.0170	7.1773	98.9	92.6
				100.0	APHCS	4	1	(3)	15.1794	0.0170	8.3844	85.7	
					PHCS	4	1	(3)	0.0391	0.0170	7.1775	98.9	85.6

Table2: The MBRs and optimal sampling plans for some selected values of a, b, a_2, C_3 ***and*** C_4 ***under APHCS and PHCS in the Type-I case***

a	b	a_2	C_3	C_4	Scheme	n^o	m^o	$(R_1^o, \cdots, R_m^o)$	T^o	ξ^o	MBR	Eff1(%)	Eff2(%)
1.5	2.0	1.0	2.0	5.0	APHCS	2	1	(1)	0.0340	0.0170	3.7787		
					PHCS	1	1	(0)	21.3863	0.0170	4.8883		129.4
2.0					APHCS	1	1	(0)	0.0888	0.0127	4.4285	85.3	
					PHCS	1	1	(0)	10.6490	0.1382	5.5777	87.6	126.0
2.5					APHCS	1	1	(0)	0.0478	0.0102	5.1695	73.1	
					PHCS	1	1	(0)	6.7469	0.4520	6.1278	79.8	118.5
3.0					APHCS	1	1	(0)	0.0169	0.1713	5.5387	68.2	
					PHCS	2	2	(2*0)	4.8399	0.7133	1.6097	303.7	29.1
3.5					APHCS	1	1	(0)	0.0145	0.6335	5.5112	68.6	
					PHCS	3	3	(3*0)	3.7380	0.7954	2.7403	178.4	49.7
4.0					APHCS	1	1	(0)	0.0127	0.2589	5.5379	68.2	
					PHCS	5	5	(5*0)	3.0297	0.7955	3.1813	153.7	57.4
5.0					APHCS	1	1	(0)	0.0102	0.1489	5.5186	68.5	
					PHCS	10	10	(10*0)	2.1826	0.7848	6.0897	80.7	110.3
1.5	0.8				APHCS	1	1	(0)	0.0136	4.7503	5.5397	68.2	
					PHCS	1	1	(0)	8.5569	1.0041	6.3410	77.1	114.5
	1.0				APHCS	1	1	(0)	0.0170	0.4695	5.5463	68.1	
					PHCS	1	1	(0)	10.6961	0.8110	6.1741	79.2	111.3
	3.0				APHCS	2	1	(1)	0.0511	0.0255	3.0255	124.9	
					PHCS	1	1	(0)	32.0771	0.0255	4.2257	115.7	139.7
	4.0				APHCS	2	1	(1)	0.0681	0.0340	2.7856	135.7	
					PHCS	1	1	(0)	42.7705	0.0340	3.9752	123.0	142.7
	5.0				APHCS	2	1	(1)	0.0851	0.0426	2.6627	141.9	
					PHCS	1	1	(0)	53.4770	0.0426	3.8508	126.9	144.6
	10.0				APHCS	2	1	(1)	0.1702	0.0851	2.5743	146.8	
					PHCS	1	1	(0)	106.9455	0.0851	3.6609	133.5	142.2
	2.0	2.0			APHCS	1	1	(0)	0.1008	0.0170	4.5823	82.5	
					PHCS	1	1	(0)	21.3921	0.4194	5.6199	87.0	122.6
		2.5			APHCS	1	1	(0)	0.0823	0.0170	4.9578	76.2	
					PHCS	1	1	(0)	21.3921	0.6614	5.8232	83.9	117.5
		3.0			APHCS	1	1	(0)	0.0582	0.0170	5.3073	71.2	
					PHCS	1	1	(0)	21.3921	0.8805	5.9730	81.8	112.5
		5.0			APHCS	1	1	(0)	0.0340	6.1106	5.5806	67.7	
					PHCS	1	1	(0)	21.3921	1.6101	6.3228	77.3	113.3
		7.5			APHCS	1	1	(0)	0.0340	6.3531	5.5806	67.7	
					PHCS	1	1	(0)	21.3921	2.3341	6.5370	74.8	117.1
		10.0			APHCS	1	1	(0)	0.0340	6.1438	5.5806	67.7	
					PHCS	1	1	(0)	21.3921	2.9403	6.6609	73.4	119.4
		1.0	0.0		APHCS	1	1	(0)	0.4109	0.0170	3.4057	111.0	
					PHCS	4	4	(4*0)	21.3921	0.4359	2.2717	215.2	66.7
			1.0		APHCS	1	1	(0)	0.1738	0.0170	3.6471	103.6	
					PHCS	1	1	(0)	21.3863	0.0170	4.1543	117.7	113.9
			3.0		APHCS	2	1	(1)	0.0340	0.0170	3.8128	99.1	
					PHCS	1	1	(0)	21.3863	0.0170	5.6223	86.9	147.5
			4.0		APHCS	2	1	(1)	0.0340	0.0170	3.8468	98.2	
					PHCS	1	1	(0)	21.3863	0.0170	6.3563	76.9	165.2
			5.0		APHCS	2	1	(1)	0.0340	0.0170	3.8808	97.4	
					PHCS	1	1	(0)	21.3863	0.0170	7.0903	68.9	182.7
			10.0		APHCS	2	1	(1)	0.0340	0.0170	4.0511	93.3	
					PHCS	1	1	(0)	21.3863	0.0170	10.7604	45.4	265.6
			2.0	10.0	APHCS	2	2	(2*0)	0.0340	0.0170	3.8111	99.1	
					PHCS	1	1	(0)	21.3919	0.0170	4.9516	98.7	129.9
				15.0	APHCS	2	1	(1)	0.0340	0.0170	3.8434	98.3	
					PHCS	1	1	(0)	21.3920	0.0170	5.0150	97.5	130.5
				20.0	APHCS	2	1	(1)	0.0348	0.0170	3.8757	97.5	
					PHCS	1	1	(0)	21.3915	0.0170	5.0783	96.3	131.0
				30.0	APHCS	2	1	(1)	0.0447	0.0170	3.9316	96.1	
					PHCS	1	1	(0)	21.3916	0.0170	5.2050	93.9	132.4
				50.0	APHCS	2	1	(1)	0.0597	0.0170	4.0179	94.0	
					PHCS	1	1	(0)	21.3921	0.0170	5.4584	89.6	135.9
				100.0	APHCS	2	1	(1)	0.0867	0.0170	4.1758	90.5	
					PHCS	1	1	(0)	21.3921	0.0170	6.0919	80.2	145.9