Modelling variables presenting long memory and nonlinearity : The GI-STAR Model

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In this endeavour, we consider two types of process :long memory and nonlinearity. In long memory processes, we can observe some features such as seasonality and time-varying dependence. Thus a kind of nonstationarity is existing. To take into account this type of phenomena, we use generalized long memory processes, the Gegenbauer processes. In the nonlinear part, we propose the STAR model introduced by Teräsvirta in 1994. We'll give the definitions of both models and their main properties before studying the new formed model,the GI-STAR.

1 The Gegenbauer processes (GI)

The gegenbauer processes are introduced in [Hosking,1981], where there are only evoked. In our model, we give the Gegenbauer processe under its canonical form :

$$(1 - 2\nu L + L^2)^{\delta} y_t = x_t$$

Conditions of stationarity and inversibility
Theorem 1 :[Gray and al,1989]
Let a Gegenbauer process with parameter ν and δ. Then
-if $|\nu| < 1$, the process is inversible if $\delta > -1/2$ and stationary if $\delta < 1/2$
-if $|\nu| = 1$, the process is inversible if $\delta > -1/4$ and stationary if $\delta < 1/4$

In the definition of Gegenbauer processes, we meet the Gegenbauer polynomials whose there come from and are thus defined :
Let $|L| \geq 1$ and $\delta \in \mathbb{R}$. We define thus for all $|\nu| \geq 1$ the Gegenbauer polynomials $C_k(\delta, \nu)$ by :

$$(1 - 2\nu L + L^2)^{-\delta} = \sum_{k \geq 0} C_k(\delta, \nu) L^k$$

We show that coefficients of development can be writen as follows :

$$C_k(\delta, \nu) = \sum_{j=0}^{[k/2]} \frac{(-1)^j \Gamma(\delta + k - j)(2\nu)^{k-2j}}{\Gamma(\delta) j!(k-2j)!}$$

where $\Gamma(.)$ representes Gamma's function. We can easily calculate these coefficients recursively by noting that :

$$\begin{cases} C_0(\delta, \nu) = 1 \\ C_1(\delta, \nu) = 2\delta\nu \\ C_j(\delta, \nu) = 2\nu(\frac{\delta-1}{j} + 1) C_{j-1}(\delta, \nu) - (2\frac{\delta-1}{j} + 1) C_{j-2}(\delta, \nu) \end{cases}$$

2 Definition of STAR model

To capture the nonlinear feature of time series, a variety of models can be used (see, Franses & van Dijk, 2000), but the most popular model is the STAR. This model has been empirically developed by Terašvirta (1994). Generally, the model STAR (p) with two regimes can be written as :

$$x_t = (\phi_{1,0} + \phi_{1,1} y_{t-1} + ... + \phi_{t-p} y_{t-p})(1 - G(s_t; \gamma, c))$$
$$+(\phi_{2,0} + \phi_{2,1} y_{t-1} + ... + \phi_{t-p} y_{t-p}) G(s_t; \gamma, c) + \epsilon_t \quad t = 1...T$$

with ϵ_t a gaussian white noise ; $G(s_t; \gamma, c)$ the transition function governing the movement from one regime to another and s_t is a variable of the transition function. Usually, it is assumed to be the logistic function :

$$G(s_t; \gamma, c) = (1 + exp - \gamma(s_t - c))^{-1}, \quad \gamma > 0$$

According to Taylor, Peel & Sarno (2001), the transition variable the most sensible is the dependent variable that is lagged one period ; the argument γ, which determines the degree of curvature of the transition function, and the argument c, which is the threshold parameter.

3 Definition of GI-STAR model

The GI-STAR model is a combinaison of Gegenbauer processes (GI) and nonlinear processes represented by STAR model :

$$\begin{cases} (1 - 2\nu L + L^2)^\delta y_t = x_t \\ x_t = \phi_1' w_t (1 - G(s_t; \gamma, c)) + \phi_2' w_t G(s_t; \gamma, c) + \epsilon_t \end{cases}$$

with

$$\phi_i = (\phi_{i,0}, \phi_{i,1}, ..., \phi_{i,p})' \quad i = 1, 2$$
$$w_t = (1, x_{t-1}, ..., x_{t-p})$$
$$\Rightarrow (1 - 2\nu L + L^2)^\delta y_t = \phi_1' w_t (1 - G(s_t; \gamma, c)) + \phi_2' w_t G(s_t; \gamma, c) + \epsilon_t$$

Before translate the lagged operator $(1 - 2\nu L + L^2)^\delta$, we must study the conditions of inversibility and stationarity.

Conditions of stationarity and inversibility :

Invertibility conditions are easiest to determine. They are the same as for the process gegenbauer. If they are different, inversibility of the gegenbuaer process would not hold. Gegenbauer process is inversible following these conditions :
-if $|\nu| < 1$, the process is inversible if $\delta > -1/2$
-if $|\nu| = 1$, the process is inversible if $\delta > -1/4$

For stationarity's conditions, we have two points :
-First, we must take into account the stationarity's conditions of the GI's part being done under some values of paramaters ν et δ evoked above following inversibility of this process.
-if $|\nu| < 1$, the process is stationary if $\delta < 1/2$
-if $|\nu| = 1$, the process is stationary if $\delta < 1/4$

It should be taken into account in this study on pain of draw wrong conclusions.
-Second, in the STAR model part, we assume that the process is stationary.

Thus, assumptions of stationarity and inversibility are :
-(H_0) :$\phi_i = \phi_{i,0} - \phi_{i,1}L - ...... - \phi_{i,p}L^p \quad i = 1, 2$, is a polynomial having all its roots outside the unit circle.
-(H_1) :$|\delta| < 1/2$ if $|\nu| < 1$ or $|\delta| < 1/4$ if $|\nu| = 1$

Considering these conditions, we obtain the infinite-order writing of model :

$$y_t = (\phi_{1,0} + \sum_{j=1}^{\infty} C_{1,j} y_{t-j})(1 - G(s_t; \gamma, c)) + (\phi_{2,0} + \sum_{j=1}^{\infty} C_{2,j} y_{t-j}) G(s_t; \gamma, c) + \epsilon_t$$

where

$$C_i(L) = \phi_i (1 - 2\nu L + L^2)^{\delta}, \quad i = 1, 2$$

with

$$C_i(L) = 1 + C_{i,1}L + C_{i,2}L^2 + ...$$

4 Estimation for GI-STAR model

In this section, we present the method for estimating parameters. We have :

$$x_t = \sum_{j=0}^{\infty} C_j y_{t-j}$$

$$\Rightarrow \sum_{j=0}^{\infty} C_j y_{t-j} = \phi_1^{'} w_t (1 - G(s_t; \gamma, c)) + \phi_2^{'} w_t G(s_t; \gamma, c) + \epsilon_t$$

$$\Rightarrow \epsilon_t = \sum_{j=0}^{\infty} C_j y_{t-j} - \phi_1^{'} w_t (1 - G(s_t; \gamma, c)) - \phi_2^{'} w_t G(s_t; \gamma, c)$$

We estimate the parameters of the model GISTAR defined by :
$\Psi = (\delta, \phi_{1,0}, \phi_{1,1}, ..., \phi_{1,p}, \phi_{2,0}, \phi_{2,1}, ..., \phi_{2,p})$. Let Ψ_0, the true value of the Ψ parameter and assume that it is inside Θ, where Θ is a compact of $\mathbb{R}^{2p+1}$.
we will use the method of conditional least squares CSS. This method is based on maximizing the sum of the conditional least squares :

$$\max_{\Psi} \sum_{t}^{\infty} \epsilon_t^2$$

We present in detail the method for estimating parameters of the GISTAR model when the noise $(\epsilon_t)_{t\in\mathbb{Z}}$ follows a Gaussian conditional distribution. The distribution of $(\epsilon_t)_{t\in\mathbb{Z}}$ conditionally to the generated tribe by the past of $(Y_t)_{t\in\mathbb{Z}}$, noted $\mathcal{F}_{t-1}$, is a distribution $\mathcal{N}(0, 1)$.

The Ψ estimator of conditional least squares CSS we will study in this part, noted $\hat{\Psi}$ in space Θ, is obtained by maximizing the conditional log likelihood of $(\epsilon_t)_{t\in\mathbb{Z}}$ on its σ-algebra $\mathcal{F}_0$. This likelihood is written in our case :

$$L(\Psi) = \frac{1}{n}\sum_{t=1}^{n} l_t \qquad with \quad l_t = -\epsilon_t^2$$

For obtain Ψ, we calculate the first derivatives of this likelihood and the information matrix associated. For all t, the first derivatives with respect to Ψ are given by :

$$\frac{\partial l_t}{\partial \Psi} = -2\epsilon_t \frac{\partial \epsilon_t}{\partial \Psi}$$

This leads us to specify the derivates of ϵ_t :

$$\frac{\partial \epsilon_t}{\partial \phi_1} = \sum_{j=0}^{\infty} y_{t-j}\frac{\partial C_{1,j}}{\partial \phi_1} + w_t(1 - G(s_t;\gamma,c))\frac{\partial \phi_1'}{\partial \phi_1}$$

$$\frac{\partial \epsilon_t}{\partial \phi_2} = \sum_{j=0}^{\infty} y_{t-j}\frac{\partial C_{2,j}}{\partial \phi_2} + w_t G(s_t;\gamma,c))\frac{\partial \phi_2'}{\partial \phi_2}$$

$$\frac{\partial \epsilon_t}{\partial \delta} = \log(1 - 2\nu L + L^2)x_t$$

To calculate the Fisher information matrix we show in completing this article it is well defined for this model, we calculate the derivative of order two following :

$$\frac{\partial^2 l_t}{\partial \Psi \partial \Psi'} = -2\epsilon_t \frac{\partial^2 \epsilon_t}{\partial \Psi \partial \Psi'}$$

The CSS estimator can then be calculated by maximizing the likelihood $L(\Psi)$.
Many other studies are underway for the enrichment of this article with a simulation that is being programmed, the existence and consistency of the estimator $\hat{\Psi}$ of CSS where many assumptions must be checked.

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