Monitoring Profiles Based on Simultaneous Confidence Bands for Penalized Spline Regression

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Statistical process control has been successfully used in a variety of industries. In most statistical process control applications, the quality of a process can be adequately represented by the distribution of a quality characteristic or of correlated quality characteristics. However, in many applications, the quality of a process or product may be better characterized by a relationship between the response variable and one or more explanatory variables; that is, the main topic is on monitoring the profile that describes such a relationship, instead of on monitoring a single quality characteristic or correlated quality characteristics. Particularly, most studies focused on the simple linear profiles. An extensive discussion of research problems on this topic has been provided by Woodall, Spitzner, Montgomery, and Gupta (2004).

Kang and Albin (2000) proposed two kinds of charting schemes for monitoring the simple linear profiles in Phase I and Phase II studies. One is a multivariate T^2 chart and the other is the combination of a exponentially weighted moving average chart and a range chart. Kim, Mahmoud, and Woodall (2003) proposed using a combination of three EWMA charts to respectively detect a shift in the intercept, slop, and standard deviation jointly in Phase II study. They also suggested using similar Shewhart-type control charts for monitoring simple linear profiles in Phase I study. Gupta, Montgomery, and Woodall (2006) compared the performance of the control charts proposed by Croarkin and Varner (1982) and Kim et al. (2003) for monitoring simple linear profiles in Phase II study. They concluded that Kim et al.'s combined EWMA charts are better than Croarkin and Varner's charting scheme. Mahmoud and Woodall (2004) investigated several control charting schemes for monitoring simple linear profiles in Phase I study. Zou, Zhang, and Wang (2006) proposed a control charting scheme based on a change point model for monitoring simple linear profiles where the process parameters are unknown but can be estimated from the in-control preliminary data. Based on likelihood ratio statistics, Mahmoud, Parker, Woodall, and Hawkins (2007) proposed a change point method for monitoring step-shifts in a simple linear profile in Phase I study. Zou, Zhou, Wang, and Tsung (2007) studied a self-starting control chart for monitoring simple linear profiles when the process parameters are unknown but some in-control preliminary data in Phase I study are available. For monitoring general linear profiles, Zou, Tsung, and Wang (2007) applied an MEWMA single chart to the transformations of estimated profile parameters in Phase II study. More studies related to monitoring linear profiles can be found in the literature. See, for example, Jensen, Hui, and Ghare (1984); Mestek, Pavlik, and Suchanek (1994); Stover and Brill (1998); Lawless, Mackay, and Robinson (1999).

In many practical applications, the profile cannot be represented adequately by a linear model. Walker and Wright (2002) and Woodall, Spitzner, Montgomery, and Gupta (2004) studied the vertical density profile which apparently can not be described by a linear profile. Williams, Woodall, and Birch

(2007) proposed three general approaches to the formulation of T^2 statistics based on nonlinear model approach in Phase I study. Colosimo and Pacella (2007) used principal component analysis to identify systematic patterns in roundness profiles. Williams, Birch, Woodall, and Ferry (2007) employed data from DuPont to monitor dose-response profiles utilized in high-throughput screening based on the nonlinear model approach of Williams, Woodall, and Birch (2007), where a four-parameter logistic regression model was used to represent the profiles. Jin and Shi (1999) applied dimension-reduction techniques to study a stamping tonnage profile, which apparently is a nonlinear profile. Lada, Lu, and Wilson (2002) and Ding, Zeng, and Zhou (2006) used dimension-reduction techniques, including wavelet and independent component analysis to study a general category of nonlinear profiles. Zou, Tsung, and Wang (2008) integrated an MEWMA procedure with a generalized likelihood ratio test (Fan, Zhang, and Zhang (2001)) based on local linear regression of Fan and Gijbels (1996) to monitor a general smooth regression profile. Qiu, Zou, and Wang (2010) proposed monitoring smooth profiles which can be described by a nonparametric mixed-effects model to account for the within-profile correlation.

In this article, a penalized spline regression model (Rupport, Wand, and Carroll (2003)) is used to represent the complex relationship between a response variable and an explanatory variable. By treating the penalized spline regression model as a linear mixed model, we can estimate the parameters of the penalized spline regression model by the method for fitting the linear mixed model. A Shewhart-type simultaneous confidence band chart for the penalized spline regression model is proposed to monitor the profiles in the Phase II study. Simulation studies are conducted to evaluate the performance of the simultaneous confidence band chart. An example used to demonstrate the applicability of the proposed chart is provided as well.

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