On Multi-Objective Decision Modelling Approach in Multivariate Surveys

Mir, A.H., Maqbool Showkat and Nazir Nageena

Sher-e-Kashmir University of Agricultural Sciences and Technology, Division of Agricultural Statistics, Shalimar, Srinagar, 191121, India

Email: ah_mir09@yahoo.com

1) Introduction

A number of real life optimisation problems can be posed as non-linear programming problems having multiple objectives. Due to the lack of suitable solution techniques, such problems are converted into a single objective problem and solved. The multi-objective optimisation methods are increasingly being utilised for solving real life problems and their innovative applications not only find wide-spread applicability of these methods but also open up new directions for research. Optimum allocation of sample sizes to various strata in univariate stratified random sampling is well known. However in real life situations more than one population characteristics are to be estimated, which may be of conflicting nature. With varying cost of measurement from stratum to stratum the cost of enumerating various characters also varies. The multivariate sampling problem was proposed as a non-linear multi-objective programming problem by Kokan and Khan (1967). A compromise allocation was suggested by Cochran (1977) for various characters, whereas Omule (1985) used dynamic programming to obtain a compromise allocation. Khan et.al (1997) used integer programming to obtain a compromise solution in multivariate stratified sampling. Jones et.al (2002) examined the allocation problem as a multi-objective optimisation problem and proposed lexicographic and e-constraint methods when complete and partial information concerning the study variables is available. Daiz et.al (2006) proposed stochastic programming approach to the allocation problem.

Multi objective decision approach is similar in many respects to the weighted linear goal programming (Ignizio 1978) method. Weighted linear goal programming (WLGP) depends on the development of weights, whereas fuzzy programming (FP) utilises a concept known as fuzzy membership function. The first step in modelling via the fuzzy linear programming method is to transform the baseline model into the vector – maximum (or minimum) model. Once the aspiration levels and degradation for each objective has been specified, we have a fuzzy model. Our next step is to transform the fuzzy model into a conventional mathematical programming model such as linear programming. Fuzzy programming approach (Kickert W.J. 1978) has been utilised recently to model and solve the multi-objective decision problems. This approach is similar to weighted linear goal programming (WLPG), differing basically in the manner in which the importance of goals are considered. WLGP depends on specification of weights whereas FP utilises the concepts of fuzzy membership function. The fuzzy programming model and solution methods are similar to the approach discussed for fitting a surface to a set of data wherein the problem is to minimise the maximum deviation of any data point to that surface (ignizio 1984).

In this paper fuzzy programming and Goal programming approach shall be used for solving the optimal allocation problems in multivariate surveys. The problem shall be formulated as a non-linear programming problem (NLPP) with multiple objective and non-linear convex constraints. The non-linearity of the constraints is handled through cutting plane technique for convex constraints. The resulting linear programming problem (LPP) with multiple objectives is then solved by fuzzy programming.

2. Allocation Problem

Suppose that p-characteristics are measured on each unit of a multivariate population which is portioned into L strata. Let n_i be the number of units drawn without replacement from the i^{th}stratum (I=1,2…L). For the j^{th}character an unbiased estimate of the population mean Y_j is Y_{jst} which has the sampling variance,

$$V_j = (\bar{y}_{jst}) = \sum_{i=1}^{L} W_i^2 S_{ij}^2 X_1 \qquad i = 1,2,3,\dots,L \;\; and \;\; j = 1,2,3,\dots,p$$

where $$W_i = \frac{N_i}{N}, \qquad S_{ij}^2 = \frac{1}{N_i - 1}\sum_{h=1}^{N_i}(y_{i\,jh} - \bar{Y}_{i\,j})^2 \qquad \text{and} \qquad X_i = \frac{1}{n_i} - \frac{1}{N_i}$$

in the usual notation.

Let C_{ij} be the cost of enumerating the j^{th} characteristic in the i^{th} stratum. The total cost of the survey, ignoring the overhead costs, is given by $Z = \sum_{i=1}^{i} C_{ij} n_i$. Let V_j be the permissible error in the estimate of j^{th} character, then the variance should not exceed V_j, i.e., $\sum_{i=1}^{L} a_{ij} X_i \le v_j$,

where $$a_{ij} = W_i^2 S_{ij}^2 \qquad \text{or} \qquad \sum_{i=1}^{L} \frac{a_{ij}}{n_i} \le b_j \qquad \text{where } b_j = v_j + \sum_{i=1}^{L} \frac{a_{ij}}{N_i}$$

Using x_i for n_i, the multivariate allocation problem can be stated as

$$\text{Minimize } Z_j = \sum_{i=1}^{L} C_{ij} x_i \;\dots(a), \qquad \text{Subject to } \sum_{i=1}^{L} \frac{a_{ij}}{x_i} \le b_j \;\dots\text{(b)}, \qquad 1 \le x_i \le N_i \;\dots\text{(c)} \qquad (1)$$

The constraints in 1(b) are convex (Kokan and Khan, 1967). The bounds 1(c) are linear and the region defined by 1(b) & 1(c) is thus convex. The problem 1(a) to 1(c) is thus a convex programming problem with linear multiple objectives.

3. Solution Using Fuzzy Programming

We first consider the problem (1) for k-th characteristic (k=1,2,…,p). This is a convex programming problem and can be solved by using any method of convex programming. However in order to be able to apply fuzzy programming technique for multiple objective problem we use the cutting plane technique of J.E.Kelly (1960). Let us write the problem for k^{th} characteristics as, Minimize $Z_k = \sum_{i=1}^{L} C_{ik} x_i$ ...(a) Subject to $g_j(X) = \sum_{i=1}^{L} \frac{a_{ij}}{x_i} - b_j \le 0$ ...(b) $1 \le x_i \le N_i$ ...(c) (2)

A starting solution for (2) is taken as a appoint that minimizes 2(a) subject to 2(c), say $X^{k(0)} = (x_1^{k(0)},..., x_L^{k(0)})$. Let a suitable tolerance limit for the convergence of the cutting plane method be denoted by $\in$ a small number and we find,

$$g_j(X^{k(0)}) = \sum_{i=1}^{L} \frac{a_{ij}}{x_1^{k(0)}} - b_j \tag{3}$$

If $g_j(X^{k(0)}) \le \in$, $X^{k(0)}$ solves the problem (2), otherwise we linearize about the point $X^{k(0)}$ as:

$$g_j(X) \approx g_j(X^{k(0)}) + \Delta g_j(X^{k(0)}) / (x - X^{k(0)} \le 0$$

where
$$\Delta g_j(X^{k(0)})' = \left[\frac{-1_{1j}}{x_1^{k(0)^2}}, \frac{-a_{2j}}{x_2^{k(0)^2}}, ..., \frac{-a_{Lj}}{x_L^{k(0)^2}} \right]$$

This gives
$$g_j(X) \approx 2\sum_{i=1}^{L} \frac{a_{ij}}{x_i^{k(0)}} - \sum_{i=1}^{L} \frac{a_{ij} x_i}{x_i^{k(0)^2}} - b_j \le 0,$$

The constraints 2 (b) are then replaced by these linearized constraints. The following LPP approximates the NLPP (2), as

Minimize $Z_k = \sum_{i=1}^{L} C_{ik} x_i$ .....(a), subject to $g_j(X) \approx 2\sum_{i=1}^{L} \frac{a_{ij}}{x_i^{k(0)}} - \sum_{i=1}^{L} \frac{a_{ij} x_i}{x_i^{k(0)^2}} - b_j \le 0,$ ......(b)

$$1 \le x_i \le N_i \quad ...(c) \tag{4}$$

Now let the solution of LPP (4) be

$X^{k(0)} = (x_1^{k(0)},..., x_L^{k(0)})$, at the t^{th} iteration we calculate,

$$gj(X^{k(i)}) = \sum_{i=1}^{L} \frac{a_{ij}}{x_i^{k(t)}} - b_j, \tag{5}$$

If
$$g_j(X^{k(t)}) \le \varepsilon \tag{6}$$

the process terminates and we have, $Xk(t) = X_{opt}\ and\ Z_k^* = (X^{k(t)})$.

If $g_j(X^{k(t)}) > \varepsilon$ for some j, then we find the most violated constraint (say h^{th})

i.e. $gh(X^{k(t)}) = Max\{g_j(X^{k(t)})]$. Relinearizing the constraint $gh(X) \le 0$ about the point $X^{k(t)}$, we have

$$gh(X) \approx g_h(X^{k(t)}) + \Delta g_h(X^{k(t)}) / (X - X^{k(t)}) \le 0.$$

This new constraint is included in problem (4) and at t^{th} iteration we solve the following LPP,

Minimize $Z_k = \sum_{i=1}^{L} C_{ik} x_i$ ....(a) subject to $2\sum_{i=1}^{L} \frac{a_{ij}}{x_i^{k(0)}} - \sum_{i=1}^{L} \frac{a_{ij} x_i}{x_i^{k(0)^2}} - b_j \le 0,$

$$2\sum_{i=1}^{L}\frac{a_{ih}}{x_i^{k(l)}}-\sum_{i=1}^{L}\frac{a_{ih}x_i}{x_i^{k(l)^2}}-b_h\le 0,\ldots.(b),\;\; 1\le x_i\le N_i \ldots.(c) \quad (7)$$

The process is then repeated until (6) is satisfied say at t^{th} iteration.

After finding t_k we solve the following LPPs for k variables,

i.e., Minimize $Z_k=\sum_{t=1}^{L}C_{ik}x_i$ ...(a) Subject to $2\sum_{i=1}^{L}\frac{a_{ij}}{x_i^{k(0)}}-\sum_{i=1}^{L}\frac{a_{ij}x_i}{x_i^{k(0)^2}}-b_j\le 0,$

$$2\sum_{i=1}^{L}\frac{a_{ih}}{x_i^{k(l)}}-\sum_{i=1}^{L}\frac{a_{ih}x_i}{x_i^{k(l)^2}}-b_h\le 0,\ldots\ldots(b),\qquad 1\le x_i\le N_i \ldots.(c) \quad (8)$$

Let the minimum values of Z_s thus found be Z_s^0 s=1, 2, …p at the corresponding minimal points. The solutions (8) have been obtained by minimizing objective functions subject to the linearized constraints. Now to find a unique solution for all the p-objective functions, denote $X_0(X_j^0)=Z_s^{jo}$, clearly $Z_s^{ko}=Z_s^0=\min Z(X_j^0)=L_s$ (say), let $\max Z_s(X_j^0)=U_s$, the difference of the maximum and minimum values of Z_s are denoted by $d_s=U_s-L_s, s=1,2,\ldots,p.$

Let δ be the dummy variable which represents the worst deviation level in a fuzzy environment (Ignizio, 1984) and therefore we have,

Minimize δ ...(a) Subject to $2\sum_{i=1}^{L}\frac{a_{ij}}{x_i^{k(0)}}-\sum_{i=1}^{L}\frac{a_{ij}x_i}{x_i^{k(0)^2}}-b_j\le 0,\;\; 2\sum_{i=1}^{L}\frac{a_{ih}}{x_i^{k(l)}}-\sum_{i=1}^{L}\frac{a_{ih}x_i}{x_i^{k(l)^2}}-b_h\le 0,$

$$\sum_{i=1}^{L}C_{ik}x_i\delta\le Zk^0\ldots..(b)\qquad 1\le x_i\le N_i \ldots.(c) \quad (9)$$

The optimum solution (fuzzy point, X_{fz}^*) of the above fuzzy LPP may not be feasible w.r.t the original non-linear constraints. In some cases the violation of certain constraints may be serious. In order to get the feasibility of such critical constraints, a heuristic approach is to move towards the negative of the gradients on the non-linear constraints.

The j^{th} constraints linearized at X_{fz}^* are

$$g_j(X_{fz}^*)\approx 2\sum_{i=1}^{L}\frac{a_{ih}}{x_i^*}-\sum_{i=1}^{L}\frac{a_{ih}x_i}{x_i^{*2}}-b_j\le 0, \qquad \text{where } X_{fz}^*=(x,\ldots,x_L^*)$$

Let $\underline{\theta}_j$ be the normal to $g_j(X_{fz}^*)$,j=1,2,….p

and let
$$\underline{\theta}=\frac{\sum_{j=1}^{p}W_j\underline{\theta}_j}{\sum_{j=1}^{p}W_j} \quad (10)$$

Where W_j is a constant defining the importance of the j^{th}constraint (e.g loss per unit in violating the j^{th} constraint). The feasible fuzzy point heuristically close to the non-feasible fuzzy point will be ,

$$X^*_{f2} = X^*_{f2} + \lambda\theta \ , \tag{11}$$

where λ is the suitable step length obtained by trial and error method

4. Chebyshev goal programming solution

The criterion behind the Chebyshev goal programming (Ignizio, 1994) is to find a solution that mimizes the single worst unwanted deviation from any goal. In other words it is a minimax goal programming approach. The conversion to the Chebyshev goal programming model of the allocation problem (1) yields the LPP, as

$$\text{Minimize} \quad \delta, \quad \text{Subject to} \quad 2\sum_{t=1x_i}^{L} \frac{a_{ij}}{x_i^{k(0)}} - \sum_{i=1x_i}^{L} \frac{a_{ij}x_i}{x_i^{k(0)^2}} - b_j \le 0, \qquad 2\sum_{t=1x_i}^{L} \frac{a_{ih}}{x_i^{k(l)}} - \sum_{i=1x_i}^{L} \frac{a_{ih}x_i}{x_i^{k(l)^2}} - b_h \le 0,$$

$$\sum_{i=1}^{L} C_{ik}x_i - \delta \le Z_k^0 \qquad 1 \le x_i \le N_i \tag{12}$$

It may be noted that in fuzzy programming model, the weights $d_s = U_s - L_s$ remain attached to δ (the dummy variable).

5. Numerical Example and Conclusion

The following numerical example illustrates the above procedure. The data in this example is from an Agricultural sample survey conducted in the District Baramulla of J & K State in India which pertains to two characters divided in two strata. The problem after simplification takes the following form,

$$\text{Min } Z_1 = 2x_1 + 3x_2 \qquad \text{Min } Z_2 = 2x_1 + 3x_2$$

$$\text{Subject to} \quad \frac{4}{x_1} + \frac{5}{x_2} \le 1 \qquad \frac{3}{x_1} + \frac{7}{x_2} \le 1 \qquad 2 \le x_i \le 20; \quad i = 1,2$$

The solution obtained by the two approaches is compared in the table below:

	Optimization w.r.t. Z_1	Optimization w.r.t. Z_2	Cheb. Point	Fuzzy Point	Feasible Cheb. Point	Feasible Fuzzy Point
Solution	$\begin{bmatrix} 7.37 \\ 6.99 \end{bmatrix}$	$\begin{bmatrix} 3.20 \\ 13.55 \end{bmatrix}$	$\begin{bmatrix} 5.93 \\ 9.25 \end{bmatrix}$	$\begin{bmatrix} 5.28 \\ 10.27 \end{bmatrix}$	$\begin{bmatrix} 8.23 \\ 10.86 \end{bmatrix}$	$\begin{bmatrix} 7.66 \\ 11.41 \end{bmatrix}$
Value of Z_1	35.70	47.05	39.61	41.37	49.04	49.55
Value of Z_2	29.10	23.14	27.04	26.11	35.55	34.39
Infeasibility of constraint (1)	0.26	0.62	0.21	0.25		
Infeasibility of constraint (2)	0.41	0.46	0.27	0.25		

The comparison of fuzzy point obtained by minimizing Z_1 yields a loss of 5.67 units in Z_1 and a gain of 2.99 units in Z_2 and with the point obtained by minimizing Z_2 yields a gain of 5.68 units in Z_1 and loss 2.97 units in Z_2.

Similarly, we can make the comparisons of the feasible fuzzy and feasible Chebshev points with the individual objective function optimization points. A final decision between using the feasible or infeasible point will naturally depend upon the value of W_1 and W_2 in comparison to the optimal objective function values. The Chebyshev point compared with the point obtained by minimizing Z_1 shows that we loose 3.91 units for Z_1 while we gain 2.06 units for Z_2 and when compared with the point obtained by minimizing Z_2 we gain 7.44 units for Z_1 while loose 3.90 units for Z_2.

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ABSTRACT

Multi-objective decision making models (Ignizio,1984) have been utilised in varied situations from emergency evacuation planning in the event of disasters to allocation of land in relation to crop planning in different regions(Sharma et,al 2007). The present paper deals with multivariate sample allocation problem which is formulated as a non-linear multi-objective decision problem (Khan et.al., 2003). The non-linearity of the constraints is handled through cutting plane technique for convex constraints. A solution procedure is developed using fuzzy programming and chebyshev goal programming approach (Ignizio, 1994). This approach provides, on comparison, an efficient and feasible solution in multivariate sample surveys. The method proposed is illustrated through a numerical illustration.