A New Test for a Unit Root in Dynamic Heterogeneous Panel Data Models

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INTRODUCTION

The unit root tests for panel data models have been widely discussed in econometrics literature in the last two decades. This paper proposes a new unit root test for dynamic heterogeneous panel data models. We here consider that each equation consists of a simple AR(1) type model with a constant term. The method proposed by Im, Pesaran and Shin (2003, hereafter IPS) is well known for the model considered here. It is an average of the Dickey and Fuller (1979, hereafter DF) t test for a unit root of an individual time series.

In this paper we introduce two modifications to the IPS method. Firstly, we subtract the initial data from the rest of data and intentionally suppress the constant term in constructing the DF t test statistic for an individual series. We call it the suppressed constant term (SCT for short) modification. Further, we call the modified IPS test in this way as the SCT-IPS test. By suppressing the constant term, we expect that the variance of the test statistic becomes smaller and consequently it gives higher power of the test.

Secondly, here we approximate the distribution of the SCT-IPS test with a normal distribution, following the pioneering work of Abadir (1995). It may be noted that IPS obtain the bias and variance correction terms for the test statistic for various values of T through extensive stochastic simulations, where T is the time series dimension. Thus, our method is much simpler and would be easier for generalization to complicated models.

Finite sample experiment shows that the distribution of the SCT-IPS test approximates a normal distribution quite well. It also shows that it is more powerful than the IPS test when the root is a local alternative, say 0.9.

The paper is organized as follows: The next section reviews the IPS model and gives modifications proposed in the paper. The third section gives some results of finite sample experiments. The fourth section gives a few concluding remarks.

MODEL AND NEW APPROACH

Review of the IPS Method

In this paper, we consider the following model:

$$(1) \quad y_{it} = \mu_i + u_{it}, \quad u_{it} = \phi_{1i} u_{i,t-1} + \varepsilon_{it}, \qquad i = 1, \cdots, N, \ t = 1, \cdots, T,$$

where $\varepsilon_{it} \sim i.i.d.(0, \sigma^2)$ and N is the cross-section dimension. We rewrite the model as

$$(2) \quad y_{it} = (1 - \phi_{1i})\mu_i + \phi_{1i} y_{i,t-1} + \varepsilon_{it},$$

or alternatively

$$(3) \quad \Delta y_{it} = (1 - \phi_{1i})\mu_i + \beta_i y_{i,t-1} + \varepsilon_{it},$$

where $\beta_i = \phi_{1i} - 1$.

IPS consider the following testing problem:

$$(4) \quad H_0 : \beta_i = 0, \ \forall i \text{ vs. } H_1 : \beta_i < 0, \ i = 1, \cdots, N_1, \text{ and } \beta_i = 0, \ i = N_1 + 1, \cdots, N.$$

They estimate (3) for each i by the ordinary least squares (OLS), and construct a test statistic by averaging t-statistic t_i for β_i of individual i

$$(5) \quad t^{IPS} = \frac{\frac{1}{\sqrt{N}} \sum_{i=1}^{N} (t_i - E(t_i))}{\sqrt{Var(t_i)}}.$$

They further show that

$$(6) \quad t^{IPS} \xrightarrow{N,T \to \infty} N(0, 1).$$

In practice, IPS estimate $E(t_i)$ and $Var(t_i)$ by simulation experiments for various values of T and compute

$$(7) \quad \hat{t}^{IPS} = \frac{\frac{1}{\sqrt{N}} \sum_{i=1}^{N} (t_i - \widehat{E(t_i)})}{\sqrt{\widehat{Var(t_i)}}},$$

and test it against the critical values of $N(0, 1)$. The experimental results in IPS show remarkably good performance of $\hat{t}^{IPS}$ even when N is very small, although their asymptotic result (6) assumes that both N and T go to infinity.

New Method: Suppressed Constant Term

Here, we intentionally suppress (or ignore) the constant term in (2) and instead estimate the following equation:

$$(8) \quad y_{it} = \phi_{1i} y_{i,t-1} + v_{it}, \qquad v_{it} = (1 - \phi_{1i})\mu_i + \varepsilon_{it}.$$

We expect efficiency gain in estimation of ϕ_{1i} by reducing the number of parameters to estimate. However, we have the problem of omitted variable. Inclusion of the constant term is equivalent to regressing $y_{it} - \bar{y}_i$ on $y_{i,t-1} - \bar{y}_{-1,i}$. Here, we use y_{0i} instead of $\bar{y}_i$. Thus, we subtract y_{i0} from both sides.

$$(9) \quad y_{it} - y_{i0} = \phi_{1i}(y_{i,t-1} - y_{i0}) + u_{it}, \ \ u_{it} = (-1 + \phi_{1i}) y_{i0} + (1 - \phi_{1i})\mu_i + \varepsilon_{it}.$$

We note that $\bar{y}_i$ and y_{i0} are both the unbiased estimator of $E(y_{it})$. Actually, the unconditional expectation of u_{it} is 0.

It is important to note that under H_0, (9) reduces to

$$(10) \quad y_{it} - y_{i0} = \phi_{1i}(y_{i,t-1} - y_{i0}) + \varepsilon_{it},$$

It is a random walk process with the initial value of zero and there is no misspecification of the model. The t-value for $\hat{\phi}_{1i}$, denoted as t_i^{SCT}, converges in distribution to the standard Dickey-Fuller distribution for a unit root test:

$$(11)\quad t_i^{SCT} \xrightarrow{T\to\infty,\ d} \left(\int_0^1 (W(r))^2 dr\right)^{-1/2} (1/2)\left([W(1)]^2 - 1\right),$$

where $W(r)$ is the standard Brownian motion. The IPS type test statistic based upon t_i^{SCT} proposed in the present paper is given by

$$(12)\quad t^{SCT-IPS} = \frac{\frac{1}{\sqrt{N}}\sum_{i=1}^{N}(t_i^{SCT} - E(t_i^{SCT}))}{\sqrt{Var(t_i^{SCT})}.}$$

It may be noted that, when H_1 is true, the process (2) is stationary and the model without constant term is misspecified. When $\phi_{1i} < 1$, the OLS estimator of $\hat{\phi}_{1i}^{SCT}$ in (9) converges to

$$(13)\quad \hat{\phi}_{1i}^{SCT} \xrightarrow{T\to\infty,\ p} \phi_{1i} + \frac{(1-\phi_{1i})(1-\phi_{1i}^2)(\mu_i - y_{i0})^2}{\sigma_i^2 + (1-\phi_{1i}^2)(\mu_i - y_{i0})^2}$$

$$(14)\quad \sim \phi_{1i} + \frac{(1-\phi_{1i})\chi_1^2}{1+\chi_1^2} \quad (\text{if } y_{i0} \text{ is normal}),$$

where χ_1^2 is the chi-square variate with a degree of freedom. Obviously, $\hat{\phi}_{1i}^{SCT}$ is not a consistent estimator of ϕ_{1i} and converges to a random variable. However, it is free from other parameters such as μ_i and σ_i^2 when y_{i0} is normal. If we assume that $0 \le (\mu_i - y_{i0})^2 < \infty$, we have

$$(15)\quad \phi_{1i} \le \text{plim}\hat{\phi}_{1i}^{SCT} < 1.$$

Since $\hat{\phi}_{1i}^{SCT}$ is inconsistent, its variance estimator is also inconsistent. Specifically,

$$(16)\quad T \times \widehat{Var(\hat{\phi}_{1i}^{SCT})} \xrightarrow{T\to\infty,\ p} (1-\phi_{1i}^2) + \frac{-2\sigma_i^2(1-\phi_{1i}^2)(\mu_i - y_{i0})^2(\phi_{1i} - \phi_{1i}^2) - (1-\phi_{1i}^2)^3(\mu_i - y_{i0})^4}{(\sigma_i^2 + (1-\phi_{1i}^2)(\mu_i - y_{i0})^2)^2}$$

$$(17)\quad \sim (1-\phi_{1i}^2) + \frac{-2\chi_1^2(\phi_{1i} - \phi_{1i}^2) - (1-\phi_{1i}^2)\chi_1^2}{(1+\chi_1^2)^2} \quad (\text{if } y_{i0} \text{ is normal}).$$

We note that $T\widehat{Var(\hat{\phi}_{1i}^{SCT})}$ also converges to a random variable, but it does not depend upon μ_i and σ_i^2, if normality of y_{i0} is assumed. Further, when $0 \le (\mu_i - y_{i0})^2 < \infty$, we have

$$(18)\quad 0 < \text{plim}T\widehat{Var(\hat{\phi}_{1i}^{SCT})} \le (1-\phi_{1i}^2).$$

Thus, the test based upon t_i^{SCT} is consistent in spite of inconsistency of $\hat{\phi}_{1i}^{SCT}$. Note that $\text{plim}T\widehat{Var(\hat{\phi}_{1i}^{SCT})}$ is smaller than the usual $(1-\phi_{1i}^2)$. It may be the result of suppressing the constant term. It is interesting to note that if $(\mu_i - y_{i0})^2$ becomes larger, $\hat{\phi}_{1i}^{SCT}$ is away from the true ϕ_{1i}, but variance estimator $T\widehat{Var(\hat{\phi}_{1i}^{SCT})}$ becomes smaller. Consequently, $\hat{\phi}_{1i}^{SCT}$ may become a more accurate estimator in the sense of mean squared error.

Normal Approximation

In this paper, we approximate the distribution of the test statistic as normal. The idea comes from Abadir (1995), Gonzalo and Pitarakis (1998), and Abadir and Lucas (2000). They show that under H_0, the asymptotic distribution of t_i^{SCT}, as described in (11), can be well approximated by a normal distribution.

We here investigate how the normal approximation works in finite samples through stochastic simulations. Here, we consider the following model:

(19) $y_{it} = y_{i,t-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim N(0,1), \quad i = 1, \cdots, T.$

We estimate $\hat{\beta}_i$ of $\Delta y_{it} = \alpha_i + \beta_i y_{i,t-1} + \varepsilon_{it}$ by the OLS and obtain the DF t-test statistic

(20) $$t_i = \frac{\hat{\beta}_i}{\sqrt{\widehat{Var}(\hat{\beta}_i)}}.$$

Note that the present model includes a constant term and t_i statistic is not exactly the same as t_i^{SCT} discussed above.

Table 1 shows our experimental results of t_i for various values of T. The number of replication is 10,000. We can see that moments of finite T are not significantly different from those of $T = \infty$ when T=25 or larger. Next, we investigate how we can approximate the distribution with a normal variate. Table 2 shows critical values of the true distribution and those derived from the approximated normal distribution adjusted by their empirical mean and standard error given in Table 1. When T=25 or larger, they are quite close each other, and it shows that the normal approximation works nicely when testing for a unit root. It effectively explains why the IPS test works well even when N is very small.

Table 1: Moments of the DF t Statistic

	$\widehat{mean}$	$\widehat{std}$	$\widehat{skewness}$	$\widehat{kurtosis}$
$T = 10$	-1.51056	1.053986	-0.35303	1.610014
$T = 25$	-1.51475	0.894556	0.10277	0.529845
$T = 50$	-1.53802	0.885599	0.108533	0.567139
$T = 100$	-1.53915	0.86726	0.198687	0.442314
$T = 250$	-1.51528	0.849224	0.222381	0.301295
$T = 500$	-1.5452	0.849533	0.209987	0.459631
$T = 1000$	-1.52833	0.84484	0.177472	0.296968
$T = 10000$	-1.52119	0.832317	0.205726	0.27437
$T = \infty$	-1.53296	0.840251	0.218155	0.334099

Note: The case of $T = \infty$ is analytical result by Nabeya (1999).

Having experimentally observed that the DF t_i statistic is well approximated by a normal variate for finite T, we try to approximate the DF t_i^{SCT} with an appropriate normal distribution. As observed in Table 1, various moments of finite T are not much different from those of $T = \infty$ when T=25 or larger. Thus, we approximate the theoretical distribution t_i^{SCT} given by Abadir (1995) when $T = \infty$, and we find that its distribution is well approximated by $N(-0.433.0.917^2)$ through extensive experiments. Consequently, we compute the test statistic for panel data as

(21) $$t^{SCT-IPS} = \frac{\frac{1}{\sqrt{N}} \sum_{i=1}^{N} (t_i^{SCT} + 0.433)}{0.917}.$$

MONTE CARLO EXPERIMENTS

In this section, we investigate the performance of $t^{SCT-IPS}$ in comparison with $\hat{t}^{IPS}$ for finite samples through Monte Carlo experiments. Here, we consider the following data generating process:

(22) $y_{it} = (1 - \phi_{1i})\mu_i + \phi_{1i} y_{i,t-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim i.i.d.N(0,1)$

Table 2: Critical values of the DF t Distribution

		Probability that t_i is less than entry				
		0.01	0.025	0.05	0.1	0.5
$T = 10$	Approximation	-3.9625	-3.5763	-3.2442	-2.8613	-1.5106
	True Values	-4.44015	-3.72955	-3.28912	-2.79648	-1.48034
$T = 25$	Approximation	-3.5958	-3.2681	-2.9862	-2.6612	-1.5148
	True Values	-3.75	-3.33	-2.99	-2.64	-1.53
$T = 50$	Approximation	-3.5982	-3.2738	-2.9947	-2.673	-1.538
	True Values	-3.59	-3.23	-2.93	-2.6	-1.55
$T = 100$	Approximation	-3.5567	-3.2389	-2.9657	-2.6506	-1.5392
	True Values	-3.5	-3.17	-2.9	-2.59	-1.56
$T = 250$	Approximation	-3.4909	-3.1797	-2.9121	-2.6036	-1.5153
	True Values	-3.45	-3.14	-2.88	-2.58	-1.56
$T = 500$	Approximation	-3.5215	-3.2103	-2.9426	-2.6339	-1.5452
	True Values	-3.44	-3.13	-2.87	-2.57	-1.57
$T = 1000$	Approximation	-3.4937	-3.1842	-2.918	-2.611	-1.5283
	True Values	-3.49069	-3.14155	-2.85919	-2.56567	-1.55477
$T = 10000$	Approximation	-3.4575	-3.1525	-2.8902	-2.5879	-1.5212
	True Values	-3.41715	-3.08915	-2.83875	-2.54805	-1.55368
$T = \infty$	Approximation	-3.4877	-3.1798	-2.9151	-2.6098	-1.533
	True Values	-3.42	-3.12	-2.86	-2.57	-1.57

Note: We calculated the true values for $T = 10, 1000, 10000$ by Monte Carlo experiment with 10,000 replications. The true values for $T = 25, 50, 100, 250, 500$ are drawn from Fuller (1976).

Here, we set $\mu_i = 500$, $\phi_{1i} = \{1, 0.9\}$, $\forall i$, $N = \{1, 10, 50\}$,$T = \{10, 50, 100\}$, the true lag order of the process is assumed to be known. The number of replication is 10,000. The experimental procedure of the SCT-IPS test is given as follows:

Step 1 Estimate ϕ_{1i} of $(y_{it} - y_{i0}) = \phi_{1i}(y_{i,t-1} - y_{i0}) + Error$ by OLS for each i.

Step 2 Compute $t_i^{SCT} = \frac{\hat{\phi}_{1i}^{SCT} - 1}{\sqrt{\widehat{Var(\hat{\phi}_{1i}^{SCT})}}}$ for each i.

Step 3 Compute $t^{SCT-IPS}$ as described in (21), and test it against $N(0, 1)$ with the significance level of 5%.

The results are given in Table 3. The empirical size of the SCT-IPS test is generally closed to its nominal size of 5%. However, it tends to be undersized when T is small and N is large, that is, when T=10 and N=50. On the other hand, the size of the IPS test is quite good for any configuration of N and T. In terms of power, the SCT-IPS test mostly dominates the IPS test. It appears that suppressing the constant term is quite effective in improving the power of the test.

The performance of the SCT-IPS test is remarkably good despite the fact that it employs the uniform adjustment factors -0.433 and 0.917. The size distortion of the SCT-IPS test when T=10 and N=50 is its only drawback. It may come from a uniform choice of the adjustment factors -0.433 and 0.917 even in this case.

Although not reported here, the power of the SCT-IPS test can be lower then the IPS test when T is large and/or ϕ_{1i} is far away from 1.0, say, ϕ_{1i}=0.5. It can be the result of model misspecification

Table 3: Size and Power Comparison of IPS Test and SCT-IPS Test

		T					
		10		50		100	
N	Test	size	power	size	power	size	power
1	IPS	0.0569	0.0639	0.049	0.117	0.0469	0.3057
	SCT-IPS	0.0488	0.0634	0.0476	0.1862	0.0515	0.4335
10	IPS	0.0583	0.097	0.0512	0.7579	0.0513	1
	SCT-IPS	0.0407	0.1305	0.0561	0.9631	0.0523	1
50	IPS	0.0585	0.199	0.0535	1	0.0498	1
	SCT-IPS	0.0193	0.302	0.0463	1	0.049	1

under H_1. It may suggest that the suppressed constant term method is valid when the alternative is closed to unity, say, 0.9 or 0.8.

CONCLUSION

This paper proposes the suppressed constant term modification to the IPS test for a unit root in panel data models. It shows that the proposed method has greater power of test than the IPS test. Another modification is the approximation of the test statistic as a normal variate. It greatly simplifies the test procedure. However, there are still some rooms for improvement in terms of the empirical size of the test in this approach. Generalizations to models with a drift term and/or higher order lags will be examined in subsequent study.

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