A Statistical Trend Model based on a Fuzzy System

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Introduction

A fuzzy trend model was proposed by Watanabe and Kuwabara (2005) and related topics were discussed by Kuwabara and Watanabe (2006). Watanabe (2008) showed that this model can be applied to general time series. However, this model has a limitation in application, since this model is developed for a long-term financial time series such as the returns of stock price indices. Therefore we propose an extended fuzzy trend model which is more suitable for application.

In this paper we use the term "trend" as the mean value function of time series. We assume that time series can be decomposed into the trend and the zero mean stationary process. In our model the trend can be either deterministic or stochastic.

The aim of this paper is to extend the former fuzzy trend model. We show applicability of the model by simulation studies and apply to real financial time series.

Fuzzy Trend Model

Let $\{y_n|n = 1, 2, ..., N\}$ denote the observed time series. The extended fuzzy trend model for $\{y_n|n = 1, 2, ..., N\}$ is given by

$$(1) \qquad y_n = \mu_n + x_n$$

$$(2) \qquad \mu_n = \sum_{k=1}^{K} \nu_k(n)\mu_n(k),$$

$$(3) \qquad R_k : \text{ If } n \text{ is } A_k, \text{ then } \mu_n(k) = \alpha(k) + \beta(k)(n - a_k), \qquad (k = 1, ..., K)$$

under the assumptions:

(A1) $\{x_n\}$ is a stationary stochastic process whose mean is zero and variance is σ_x^2,
(A2) $u(k) = (\alpha(k),\ \beta(k))'$ is the bivariate unobserved deterministic or stochastic process, and
(A3) the joint distribution of $\{x_n|n = 1, ..., N\}$ is the same as the conditional distribution of $\{x_n|n = 1, ..., N\}$ for given $\{u(k)|k = 1, ..., K\}$,

where R_k is the fuzzy if-then rule and ν_k is the membership function of the fuzzy set A_k. a_k is the center of the fuzzy set A_k, which is defined later. On fuzzy set A_k we assume that

(A4) the entire set of A_k is the set $\{t|0 \leq t \leq N_{\max}\}$, where $N \leq N_{\max}$, and the membership functions ν_1,..., ν_K satisfy the equation:

$$(4) \qquad \sum_{k=1}^{K} \nu_k(t) = 1 \quad \text{for} \quad 0 \leq \ \forall t \leq N_{\max}.$$

If $\beta(k) = 0$ for all k, then the above system becomes the former model.

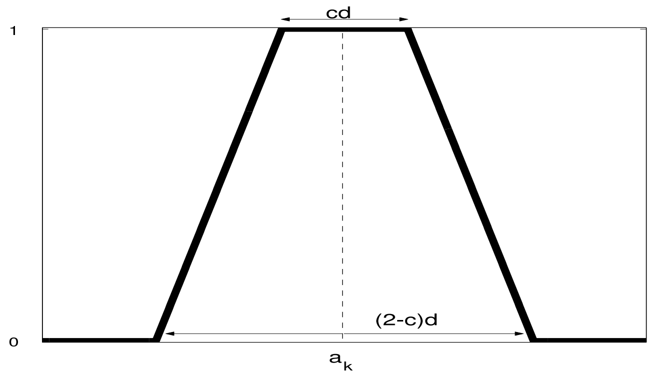

Figure 1: Membership function ν_k.

The input-output system given by (2) and (3) is a kind of the Takagi-Sugeno's fuzzy system (Takagi and Sugeno (1985)), which is well-known in the fuzzy control theory. The output is the trend $\{\mu_n\}$ and the input is the unobserved process $\{u(k)\}$. The output $\{\mu_n\}$ is given by a weighted sum of the input and weights are determined by membership values. For given $\{u(k)\}$ the first term $\{\mu_n\}$ is the conditional mean function of $\{y_n\}$. When $\{u(k)\}$ is stochastic, $\{\mu_n\}$ and $\{x_n\}$ are independent from the condition (A3).

The model given by (1)–(3) can be represented by a regression form

(5) $$y = Bu + x,$$

where $y = (y_1, ..., y_N)'$, $x = (x_1, ..., x_N)'$, $u = (u'(1), ..., u'(K))' = (\alpha(1), \beta(1), ..., \alpha(K), \beta(K))'$ and $N \times 2K$ matrix B is determined by ν_k as follows:

$$\begin{aligned} (n, 2k-1)-\text{element of} \quad B \quad &\text{is } \nu_k(n), \\ (n, 2k)-\text{element of} \quad B \quad &\text{is } \nu_k(n)(n - a_k) \end{aligned}$$

for $n = 1, ..., N$ and $k = 1, ..., K$. However, B is unknown differently from the usual regression model (see Grenander and Rosenblatt (1957) and Vogelsang (1998), for example). Note that the width of B is also unknown.

In this paper we use the membership function ν_k which has the form shown in Fig. 1, where the width parameter d is an integer larger than one, $a_k = (k-1)d$ and the shape parameter c satisfies $0 \leq c \leq 1$.

In this study we consider three cases: (i) $c = 1.0$ (rectangular/clisp), (ii) $c = 0.5$ (trapezoid) and (iii) $c = 0.0$ (triangle).

The feature of the fuzzy trend model is the existence of the latent process $\{u(k)|k = 1, ..., K\}$. The analysis on the trend can be achieved by investigating the latent process $\{u(k)\}$. We can derive useful information from the series $\{\alpha(k)\}$.

Identification

The number of rules or length of the process $\{u(k)\}$ can be determined from N by the equation

(6) $$K = \left[\frac{N + d - 1}{d}\right] + 1,$$

where $[t]$ means the minimum integer that is greater than or equal to t, for $2 \leq d \leq N$. For $d > N$ we set $K = 1$, which implies that μ_n is constant. The integer d is unknown and then K is also unknown.

Under the above setting indemnification is equal to determination of c and d from the observed series $\{y_1, ..., y_N\}$. For given c and d the latent process $\{u(k)\}$ can be estimated by the least squares method, since B is fixed. The least squares estimator of u for fixed d is given by $\hat{u} = (B'B)^{-1}B'y$.

If the joint distribution of $\{x_1, ..., x_N\}$ is parameterized, the maximum likelihood method can be applied for estimation of its parameters and u simultaneously. And if we can assume that numbers of possible values of d and c are finite respectively, an information criterion such as AIC or BIC can be defined. Then the model can be identified by using an information criterion.

When the trend structure is unknown, however, the structure of $\{x_n\}$ is also unknown generally. In this case the quasi AIC can be defined by assuming $\{x_n\}$ is the Gaussian white noise and using the least squares estimate of $\{u(k)\}$. At the first stage of time series analysis such an approach might be efficient.

On the other hand applying the model for specified d will derive useful information. For daily time series $d = 5 \sim 7$ or $d = 25 \sim 31$ corresponds to one week or one month respectively. In this case it might be convenient to fix c to 0.5.

Simulation

We achieve simulation studies on identification using AIC and BIC. For identification of the former model BIC is recommended, since AIC tends to select very small d. On the other hand for our model, results by AIC and BIC are not so different. We omit the detail of the results by simulation studies.

Application

We apply the former and proposed models to time series of daily exchange rates for US Dollar against Japanese Yen (8/3/2006-9/1/2010). Original series is plotted in Fig. 2.

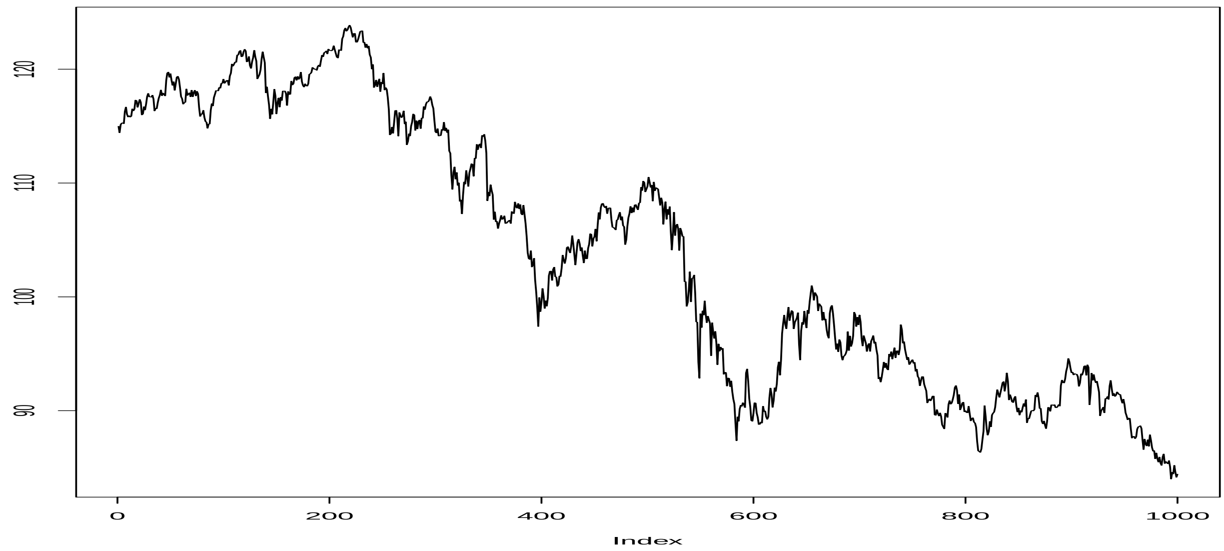

Figure 2: Time series of exchange rates

Values of d and c are identified as 9 and 0.0 for the proposed and former models by using the quasi BIC. Estimated trends are illustrated in Figs. 3 − 5. By the quasi AIC the proposed model is identified as $d = 9$ and $c = 0.0$ identically to quasi BIC. On the other hand the quasi AIC selects $d = 3$, which is the minimum value of d given previously. It is found that the proposed model is more suitable than former model from Figs. 3 − 5.

Figs. 6 − 9 show the estimated trends by the proposed models for $d = 50,\ 75$ with fixed $c(= 0.5)$. We can apply our model similarly to the moving average method.

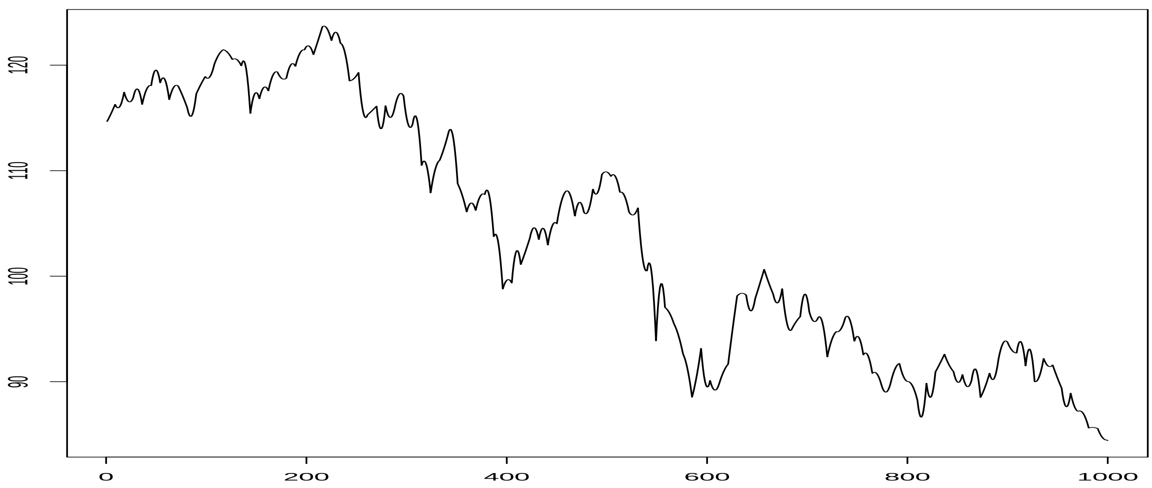

Figure 3: Estimated trend – new model ($d = 9, c = 0.0$)

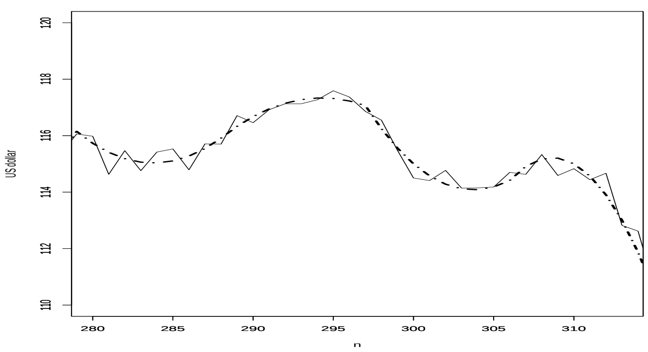

Figure 4: Estimated trend (part) – new model ($d = 9, c = 0.0$)

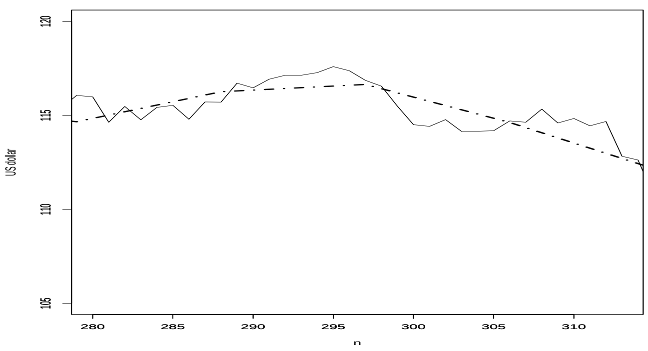

Figure 5: Estimated trend (part) – former model ($d = 9, c = 0.0$)

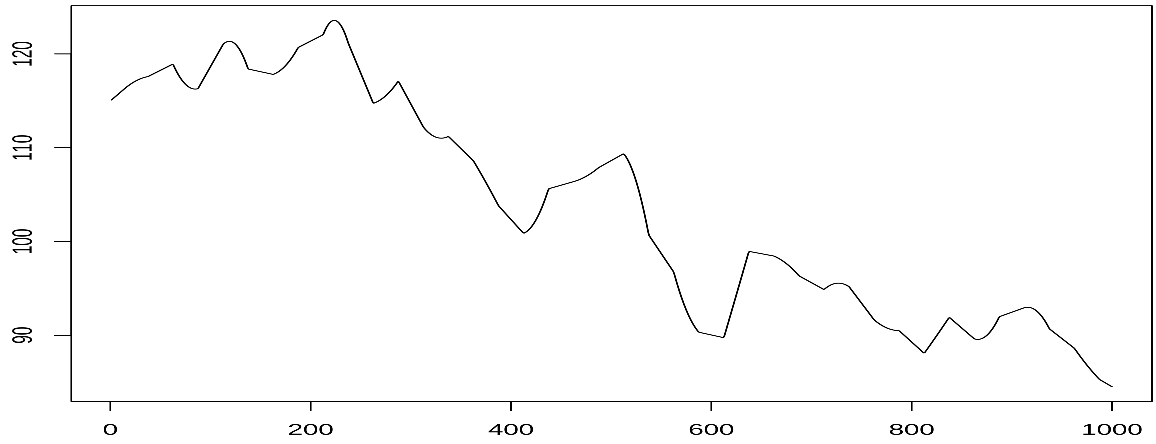

Figure 6: Estimated trend – new model ($d = 50, c = 0.5$)

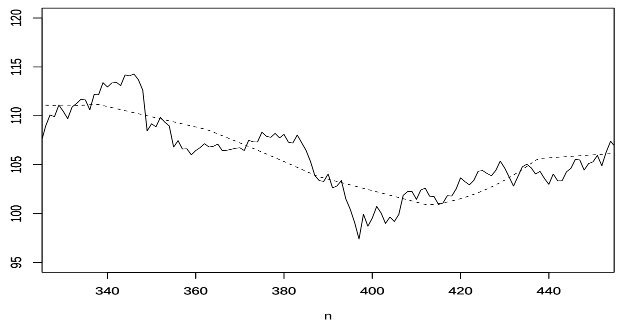

Figure 7: Estimated trend (part) – new model ($d = 50, c = 0.5$)

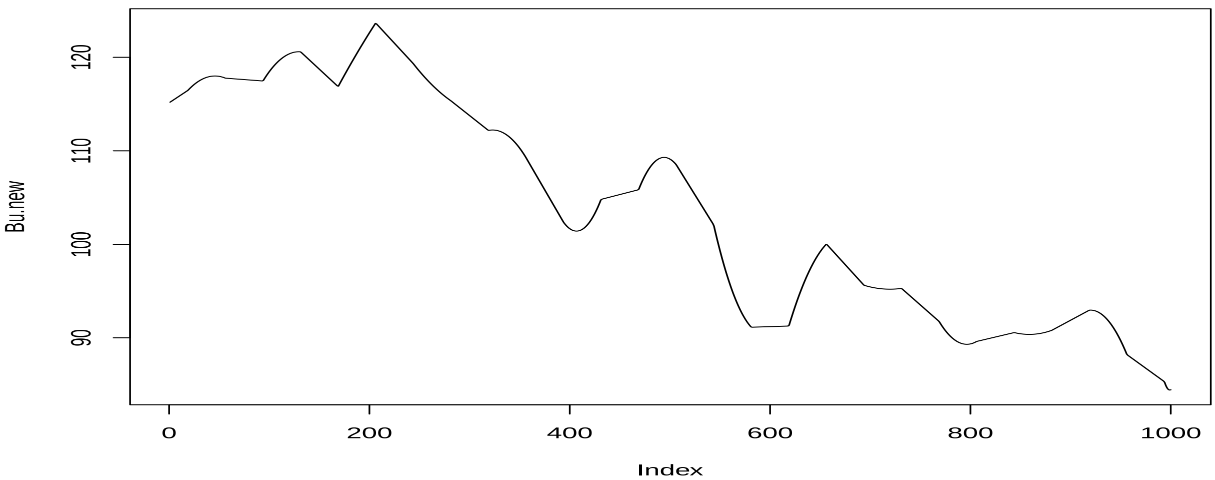

Figure 8: Estimated trend – new model ($d = 75, c = 0.5$)

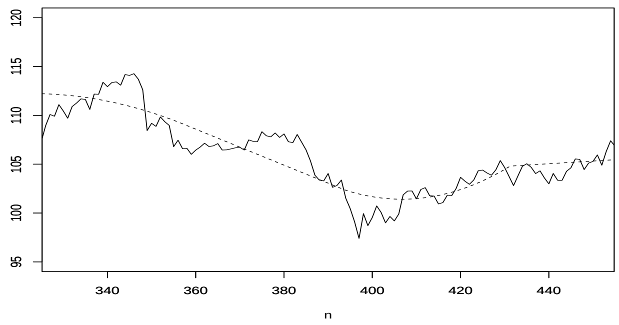

Figure 9: Estimated trend (part) – new model ($d = 75, c = 0.5$)

Conclusion

In this paper we extend the fuzzy trend model for trend analysis. The simulation studies and application show applicability of the proposed model. The fuzzy trend model provides an alternative approach for trend analysis.

However simulation studies are limited. Further studies are required for various cases. Especially the identification problem should be considered in detail.

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RÉSUMÉ (ABSTRACT)

An extended fuzzy trend model is proposed for analyzing trend of time series. This model is a simple time series model based on a fuzzy system composed of fuzzy if-then rules. Applicability of the model is shown by some examples and simulation studies.