

The Post Enumeration Survey of the 2010 Census, an Experience of Indonesia

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1. Background

Statistics Indonesia conducted a Population Census in May 2010. For the purpose of enumeration, the country was divided into 724,052 census blocks (CBs), each with an average of 100 households. The exercise involved approximately 530,876 enumerators. The 2010 Population Census of Indonesia is a major source of population data by geographical region, demographic composition and other features. This massive exercise required a huge amount of resources and challenging efforts to maintain the quality of the operation.

The data collected through any field inquiry is subject to a certain amount of error due to mistakes in the field by the enumerators and/or respondents. A complicated operation like the population census is no exception; some amount of error is inevitable considering the fact that a large number of enumerators and supervisors are engaged in the data collection.

Evaluation of the quality of the data should be an integral part of any population census programmes. One of the methods used to evaluate the results of a population census is to conduct a Post Enumeration Survey (see Appendix for PES methodology). Statistics Indonesia conducted a Post Enumeration Survey (PES) for the 2010 Population Census of Indonesia in July 2010. The planning and management of the PES was undertaken by the Directorate for Development of Census and Survey of the Statistics Indonesia; UNFPA provided technical assistance to invite a consultant on sampling design for the PES.

2. Quality Improvement and Evaluation of 2010 Population Census

Post Enumeration Survey

The planning, implementation and adherence to proper methodology of the PES for the 2010 Population Census was much better than for any PES conducted in Indonesia in the past. Compared to previous post enumeration surveys, the 2010 PES received sufficient financial and technical resources from the government. Consequently, Statistics Indonesia was able to set an objective to provide estimates of census coverage errors at the provincial level. In addition the technical personnel of Statistics Indonesia were committed to implementing strict technical requirements for a sound PES.

While the sample size of the 2000 PES was only 40 CBs in 4 provinces (11 sub districts/cities) the sample size of the 2010 PES was 1200 CBs in each of the 33 provinces. This sample size was necessary to meet the objective of the 2010 PES to provide province level estimates for coverage and content errors. It should also be noted that for the first time Statistics Indonesia conducted field reconciliations for the questionable matches between the characteristics collected in the 2010 Census and those collected in the PES.

Census Quality Assurance

Statistics Indonesia conducted a “census quality assurance” field operation during the first week of enumeration. Supervisors were sent from headquarters to 351 districts/cities in all 33 provinces to supervise the enumerator training, standard operational procedures (SOP), and field enumeration. The results of this supervision were used as inputs for the BPS district/city management to correct the mistakes in the field as soon as possible to improve the quality of the census.

The supervisors were Statistics Indonesia staff, echelon IV and echelon III, as well as students and lecturers of the statistics college in Jakarta. They were assigned to go to the field to supervise the implementation of the SOP by using a short questionnaire. The supervisors were instructed to send the results of their observation via text messages through the SMS-gateway server in BPS headquarters. This enabled the Statistics Indonesia management to directly monitor through their cell phones/blackberries the progress of the quality assurance field operation as well as the impact of the on-the-spot decision-making of the district/city managers.

Pilot of the 2010 PES

In preparation for the 2010 Population Census, Statistics Indonesia conducted two dress rehearsals about one year before the census date:

- 1st dress rehearsal was held in 2 provinces: DKI Jakarta and Nusa Tenggara Barat;
- 2nd dress rehearsal was held in 5 provinces: Sumatera Utara, Sumatera Selatan, Jawa Barat, Sulawesi Tenggara and Papua Barat.

Following each of the above-mentioned census dress rehearsals, a pilot of the 2010 PES was also conducted. The 1st pilot was held in June 2009, and the 2nd was held in October 2009 in the same provinces as the census dress rehearsals. Statistics Indonesia was fortunate to have the services of Mr. David J. Megill, a UNFPA consultant, who provided recommendations for significant improvement in the development of the PES methodology and questionnaire design, in such a manner that the objectives of the PES could be achieved. David Megill visited Jakarta when the first PES pilot was in the field, and his inputs were incorporated into the methodology and questionnaire design for the second PES pilot. The two Pilots provided valuable experience and insights to improve the planning of the overall field methodology, organization, the matching and reconciliation rules, as well as the PES analytical procedures. The pilots also provided input into the design of the logistical flow of the documents between the PES and the census so that independence of the two sources of data could be strictly maintained.

3. PES Sample Design

The PES for the 2010 Population Census was conducted in July which was two months after the census in a scientifically-designed probability selected sample of 1,200 census blocks (CBs) in all 33 provinces of Indonesia for provincial level estimation. Only individual characteristics were collected from household members living in ordinary households for the PES. The allocation of the 1,200 sample CBs among the 33 provinces was made proportional to the square root of the population size. The resulting sample allocation was further adjusted to have a minimum of 24 CBs and a maximum of 80 CBs per province.

The sampling frame used for selecting the sample was the list of CBs arranged by the number of households resulting from the listing prepared during the census.

The sample was selected using 2-stage sampling:

- 1st stage: select a number of sub districts in each province using a systematic sample with probability proportional to size (PPS), where the size was the number of households in the sub district (from the listing of the census).
- 2nd stage: select a number of CBs in the selected sub districts also using a systematic sample with probability proportional to size, where the size was the number of households in the CB (from the listing of the census).

Before the second stage sample selection was carried out, the CBs in the selected districts were sorted by urban and rural characteristics. As a result, the 1200 CBs selected were located in 392 sub districts in 316 districts/cities in the 33 provinces.

The sampling weight applied to a given CB was inversely proportional to the probability of selection of the CB. The weight for the j -th CB in the i -th sub district of a province is calculated as:

$$w_{ij} = \frac{Z_0}{nm_i z_{ij}},$$

where : $Z_0 = \sum_i^N \sum_j^{M_j} z_{ij}$

i = index of sub district,

j = index of CB,

n = number of selected sub districts in a province,

m_i = number of selected CBs in i -th sub district in a province,

z_{ij} = number of households in the j -th CB, i -th sub district in a province,

N = total number of sub district in a province,

M_i = total number of CBs in the i -th subdistrict,

Z_0 = total number of households (from the listing of the census) in a province.

Since all households within the selected CB were enumerated, the same weight is applied to each household and individual within the CB.

Individual characteristics collected in the PES were:

- | | |
|--|---|
| 1. Name | 7. Marital status |
| 2. Relationship to the head of household | 8. Number of children still and living within the household |
| 3. Sex | 9. Number of children still living and living outside the household |
| 4. Age | 10. Number of children born alive who have died. |
| 5. School attendance | |
| 6. Highest level of education attended | |

4. Fieldwork

The enumerators were divided into teams of three, each with a team leader. The sample of the population were interviewed by using the PES questionnaire. Good quality CB maps were generally available for use for the PES field operation in all provinces except Papua.

Strategies implemented to maintain independence between the census and the PES were as follows:

1. The best census enumerators and supervisors should be used.

2. All PES enumerators and supervisors should be assigned to work in areas other than those in which they worked during the census.
3. The results of the census enumeration in the selected PES areas should be kept confidential from the PES field workers in those areas.
4. All census materials from the selected PES areas should be collected and sent to the provincial offices before the PES enumerators go into the field.

5. Matching Procedure, Field Reconciliation and Data Processing

“The two-way matching between the PES and the census questionnaires is one of the most critical operations that will affect the quality of the PES coverage evaluation. At the same time matching is one of the most difficult tasks, so it is necessary to ensure that sufficient resources are allocated to training and quality control for the matching operation¹.”

The reconciliation visit, if necessary, following the matching operation is an integral part of the PES methodology. During the field reconciliation it was possible to follow up each household and person enumerated in the census but not in the PES, in order to determine if the persons were correctly or erroneously enumerated in the census. The reconciliation visit was also used to follow up cases of possible matches so that the final match status could be determined.

The use of census building stickers for most households made the matching between the PES and census questionnaires easier and less time-consuming.

The matching process was done through many stages:

1. Desk-matching carried out at the provincial statistical offices, supervised by the matching supervisor from headquarters;
2. Case-by-case matching for the unmatched records (P-sample – see appendix) with data from the Census at headquarters;

Data capture was carried out using the FoxPro software. Stata was used to compile and generate the initial tables. The initial tables were exported to Microsoft Excel in order to produce the final tables for the report.

Some of the challenges in matching, data processing and field reconciliation were:

1. There were difficulties in matching names, where individuals reported different names;
2. In many rural areas unique physical addresses were not available for comparing individuals and households;

6. Analysis of the PES

Coverage analysis

The PES involved using dual system estimation procedures for measuring the census coverage errors. This estimation procedure is based on the assumption of independence between the census and the PES enumeration. A two-way matching between the households and persons enumerated in the census and the PES was used to identify the matched persons enumerated in both, as well as the persons enumerated only in the PES or only in the census.

¹ David J. Megill, “Review of Sample Design and Estimation Methodology for Post Enumeration Survey to Evaluate the 2010 Indonesia Population Census” (2009), page 9.

The matching was limited to non-movers and out-movers, that is, the persons who were supposed to be enumerated in the household at the time of the census. The PES population also includes in-movers, who became part of the household after the census enumeration. To estimate the number of matched movers it involves applying the match rate for out-movers to the number of in-movers. The PES questionnaire was designed to determine the moving status of each person enumerated in the PES, and to enter the appropriate matching status during the matching operation or following the field reconciliation.

*Table 1. Estimates of the Coverage Rate for Persons by Province**

Province	Coverage rate	Omission rate	Erroneous inclusion rate	Gross coverage error rate	Net coverage error rate
(1)	(2)	(3)	(4)	(5)	(6)
[11] Nanggroe Aceh Darussalam	95.30	4.70	2.10	6.775	2.726
[12] Sumatera Utara	96.90	3.10	1.90	4.981	1.139
[13] Sumatera Barat	92.00	8.00	4.80	12.547	3.378
[14] Riau	89.90	10.10	4.10	13.914	6.213
[15] Jambi	93.00	7.00	2.80	9.802	4.365
[16] Sumatera Selatan	93.50	6.50	1.20	7.393	5.427
[17] Bengkulu	93.50	6.50	3.30	9.742	3.373
[18] Lampung	93.70	6.30	3.20	9.281	3.198
[19] Bangka Belitung	93.00	7.00	1.70	8.590	5.372
[21] Kep. Riau	86.30	13.70	6.50	18.721	7.616
[31] DKI Jakarta	88.60	11.40	4.20	15.288	7.576
[32] Jawa Barat	95.10	4.90	1.60	6.435	3.406
[33] Jawa Tengah	95.00	5.00	2.60	7.560	2.463
[34] DI Yogyakarta	94.10	5.90	2.40	8.369	3.534
[35] Jawa Timur	95.10	4.90	2.10	6.996	2.883
[36] Banten	91.50	8.50	3.20	11.582	5.430
[51] Bali	92.50	7.50	2.60	9.914	5.003
[52] Nusa Tenggara Barat	93.30	6.70	1.90	8.615	4.842
[53] Nusa Tenggara Timur	92.40	7.60	2.20	9.300	5.593
[61] Kalimantan Barat	94.00	6.00	2.70	8.457	3.357
[62] Kalimantan Tengah	92.70	7.30	2.80	9.845	4.630
[63] Kalimantan Selatan	93.70	6.30	2.60	8.849	3.779
[64] Kalimantan Timur	89.70	10.30	1.90	12.010	8.491
[71] Sulawesi Utara	91.70	8.30	4.00	12.343	4.524
[72] Sulawesi Tengah	95.80	4.20	1.60	5.806	2.655
[73] Sulawesi Selatan	95.10	4.90	4.50	9.273	0.355
[74] Sulawesi Tenggara	93.80	6.20	5.00	10.995	1.278
[75] Gorontalo	95.40	4.60	1.20	5.742	3.411
[76] Sulawesi Barat	96.80	3.20	2.30	5.321	0.863
[81] Maluku	94.50	5.50	1.40	6.624	4.138
[82] Maluku Utara	91.60	8.40	1.80	9.767	6.630
[91] Papua Barat	90.00	10.00	2.50	11.994	7.722
[94] Papua	83.50	16.50	7.50	20.147	9.765
Total	94.00	6.00	2.60	8.495	3.558

*subject to rounding error; data per May 6, 2011

Table 1 above shows that the national coverage rate was 94.00 per cent, while the omission rate, which was the ratio of the difference between the PES population and the matched population to the PES population, was 6.00 per cent. The erroneous inclusion rate, which was computed from the ratio of the erroneous inclusions to the census population stood at 2.60 per cent. The gross coverage rate, which was calculated from the ratio of the sum of omissions and erroneous inclusions to the census population, was found to be 8.495 per cent. On the other hand, the national net coverage error rate, which was computed from the ratio of the difference between the preliminary dual system estimate of true population and the census population to the preliminary dual-system estimate of true population, was 3.558 per cent. The provincial figures showed that the lowest coverage rate was 83.50 percent in Papua, while the highest coverage rate was 96.90 percent in Sumatera Utara. The lowest net coverage error rate was 0.355 per cent in Sulawesi Selatan, while the highest was 9.765 per cent in Papua.

Content Analysis

The content error analysis was based on the data for matched persons (non-movers and out-movers). The content error tabulations consist of cross-tabulating the PES responses for each item with the corresponding census responses. Summary statistics for content error analysis include the index of inconsistency for each category, the aggregate index of inconsistency, and the corresponding 95 per cent confidence intervals, as well as the net difference rates and the rate of agreement.

Content error, also known as response error, is defined as the deviation of the obtained value from the true value for a given characteristic. Response error can be divided into response bias (systematic error) and response variance (variable error). The PES is regarded as a replication, an independent re-interview of a sample from the census for the purpose of estimating variable error, not bias. The PES content error analysis measures **consistency**, not which answers are right or wrong, i.e., it measures how **differently** answers are reported between the census and the PES². All characteristics mentioned earlier under the section 3 were included in the content error analysis with the exception of the respondent's name, school attendance, number of children (male) still and living within the household, and number of children born alive who have died.

To ensure comparability between the PES and the census, the same wording was used on the questionnaire and in the interview, response and pre-code categories, and also the same concepts and definitions of characteristics were maintained in the PES. First, estimated totals from the census and the PES were compared for matched persons for the selected characteristics. Variability between the census and the PES is then measured by means of four different indicators: the net difference rate, the index of inconsistency (simple and aggregate), the gross difference rate, and the rate of agreement. This paper only presents Aggregate Index of Inconsistency for the selected characteristics.

Net Difference Rate (NDR). The net difference rate is the difference between the number of cases in the census and the number of cases in the PES that fall under each response category, relative to the total number of matched persons in all response categories.

Index of Inconsistency. The index of inconsistency is the relative number of cases for which the response varied between the census and the PES. It is the ratio of the simple

² Statistics South Africa, "Post Enumeration Survey: Results and Methodology" (2004), page 25.

response variance to the total variance of the characteristic, including its variability in the population. It is calculated for each response category.

Gross Difference Rate (also Off-Diagonal Proportion). The gross difference rate (GDR) is calculated for the characteristic as a whole. It is the number of discrepancies between the census responses and the PES responses relative to the total number of persons matched. It is equivalent to the sum of all cells off the diagonal, for all categories, or the complement of the sum of the diagonal cells.

Rate of Agreement. The rate of agreement is the complement of the gross difference rate. A low rate of agreement indicates a high degree of variability, and vice versa.

Standards for the interpretation of the different content error measures

Measure	Low	Moderate	High
Index of inconsistency	<20	20 – 50	>50
Aggregate index of inconsistency	<20	20 – 50	>50
Absolute value of NDR relative to mean or proportion (NDR/P)	<0.01	0.01 – 0.05	>0.05

Source: "Post Enumeration Survey Operational Guidelines Technical Report", New York, April 2010.

Table 2 below shows that at the provincial level, there is low variability between the census and the PES for some characteristics, such as relationship to the head of household, sex, age, marital status, number of living children (female) who live within the household. There is moderate variability for characteristics such as highest level of education attended, and number of living children who live outside the household (male, female).

Table 2. Aggregate Index of Inconsistency by Characteristics and Province**

Code	Province	Relation	Sex	Age	Education	Marital	F_in	M_out	F_out
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
11	Nangroe Aceh Darussalam	8.88	2.47	21.80	37.60	10.62	13.71	25.50	26.96
12	Sumatera Utara	5.13	3.02	15.64	29.78	9.10	13.16	24.31	26.10
13	Sumatera Barat	15.47	2.25	19.70	37.69	9.56	10.79	20.03	21.01
14	Riau	6.90	2.66	18.51	33.01	9.48	10.97	27.82	28.35
15	Jambi	11.72	2.35	20.01	37.10	8.93	12.00	26.49	28.05
16	Sumatera Selatan	7.46	2.48	18.68	29.11	9.03	9.83	18.15	20.01
17	Bengkulu	9.14	4.07	25.29	37.82	8.16	12.79	19.69	21.79
18	Lampung	10.04	3.07	23.48	39.56	10.49	13.24	28.68	29.24
19	Kep. Bangka Belitung	10.02	2.45	17.30	32.72	7.65	9.16	18.99	20.59
21	Kep. Riau	11.37	3.58	17.28	34.80	12.95	10.87	27.68	24.66
31	DKI Jakarta	12.55	2.84	15.36	38.63	11.28	10.21	33.95	35.55
32	Jawa Barat	10.93	3.11	21.13	35.54	10.17	12.08	26.00	26.17
33	Jawa Tengah	9.61	1.97	16.02	37.5	7.58	10.94	25.54	25.01
34	D.I. Yogyakarta	11.26	1.94	14.81	35.15	7.37	8.20	18.41	18.83
35	Jawa Timur	10.78	2.09	18.38	35.92	10.00	11.54	23.52	24.70
36	Banten	11.46	2.92	21.50	36.04	11.16	12.94	28.97	30.29
51	Bali	13.75	2.17	19.57	35.49	9.19	9.09	25.12	22.90
52	Nusa Tenggara Barat	12.90	3.48	35.09	37.95	11.50	13.62	22.78	24.65
53	Nusa Tenggara Timur	14.48	4.39	22.27	36.99	12.50	14.01	26.71	28.13
61	Kalimantan Barat	9.80	2.64	19.65	41.40	11.08	13.33	28.03	27.19
62	Kalimantan Tengah	12.82	3.91	23.74	40.82	11.41	13.18	26.71	29.08

Code	Province	Relation	Sex	Age	Education	Marital	F_in	M_out	F_out
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
63	Kalimantan Selatan	10.09	1.84	19.14	32.66	8.76	9.37	20.68	20.14
64	Kalimantan Timur	11.37	3.79	15.68	34.00	10.39	12.42	31.51	27.29
71	Sulawesi Utara	13.84	2.92	11.26	38.39	10.09	12.43	22.50	23.43
72	Sulawesi Tengah	12.02	3.37	20.75	34.66	9.69	12.21	27.41	26.60
73	Sulawesi Selatan	11.65	3.17	22.69	36.87	10.67	11.70	23.03	25.23
74	Sulawesi Tenggara	11.97	2.17	23.68	32.86	9.17	11.10	24.22	25.95
75	Gorontalo	10.01	2.55	18.32	36.10	8.21	13.02	21.24	22.45
76	Sulawesi Barat	10.52	2.58	22.20	35.63	9.26	10.00	23.85	22.88
81	Maluku	14.15	3.57	21.39	33.36	11.65	13.79	26.59	28.47
82	Maluku Utara	11.93	2.87	19.33	31.81	8.36	11.22	22.60	22.95
91	Papua Barat	15.84	3.92	19.6	34.39	12.35	14.76	29.52	33.58
94	Papua	12.63	6.03	40.13	45.04	20.85	18.79	36.09	37.17
Total		10.87	2.85	19.95	35.38	9.95	11.84	24.73	25.50

**Subject to rounding error; data per May 6, 2011

7. Lessons Learned & Way Forward

It is reported that PES was conducted two months after the Census because the Census enumeration was extended for 2 weeks, and by then the month of Ramadhan, the fasting month, was approaching which ends with „Idul Fitri, a major Islamic holiday celebrated by the majority of the population; therefore the fieldwork could not be conducted until later in the year. This meant that the implementation of the PES was delayed, resulting in the fieldwork being conducted two months after the census enumeration day. To avoid large distortion of the population structure at the time of PES enumeration, the field work should be carried out immediately after the census.

The supervisors of the desk-matching were national instructors of the PES. Most of them were not staff of the Directorate of Methodology – subject matter of PES and were not involved in PES from the beginning. Therefore, they were not too concerned, dedicated and motivated to the job since they did not understand the importance and the philosophy of PES very well.

A major benefit of conducting the PES is the enormous experience that personnel of Statistics Indonesia have gained by conducting the PES themselves. They know the challenges in planning the exercise, were exposed to problems of fieldwork after a census, have been trained in data matching procedures, have implemented the matching procedures, have participated in the field reconciliation, and have done the data processing, tabulation, analysis and write up. All these experiences have contributed to the huge institutional memory of Statistics Indonesia. These will enable Statistics Indonesia to conduct a better PES in the future.

8. Constraints and Limitations of the 2010 PES

There were several constraints and limitations:

1. The PES was conducted at the same time as the census was being processed. Therefore, the district and provincial offices were very busy at the time the PES was conducted.
2. The design of the PES, especially the matching and reconciliation stages, were complex and needed highly-experienced, dedicated and motivated personnel to carry it out efficiently and effectively.

3. Until this paper was written, the final counts of the 2010 population census have not been released yet by Statistics Indonesia. This paper was written using census figures per May 2011.
4. This paper only presents Agregate Index of Inconsistency for the content error analysis because of the limitation of page number.

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Appendix

Objectives

A Post Enumeration Survey (PES) provides a concrete statistical basis for estimating census coverage and content error. It also provides an evaluation of the reliability of some of the characteristics reported in the census. The specific objectives of the PES are:

1. To estimates household and population coverage error;
2. To measures content error of individual characteristics;
3. To assists data users in using the census data judiciously by giving them greater insight into the quality and the limitations of the data;
4. To assists in providing a better basis for demographic projections

The activities of the PES includes:

- | | |
|--|---|
| 1. Preparatory activities | 4. Enumeration for every household and individual in the selected CBs |
| 2. National instructors and field enumerators training | 5. Desk-match for households and persons |
| 3. Field orientation using census block map to know the working area | 6. Field reconciliation |
| | 7. Data processing |
| | 8. Estimation of coverage and content error |
| | 9. Report preparation and dissemination |

Methodology

A PES is an independent survey that replicates a census in sampled areas. The PES and census records are then matched (compared item by item) in terms of households, individuals in the households and their characteristics. The results of the comparison are used to measure the coverage and content errors.

The PES actually involves two samples, namely, the P-sample and the-E sample. The "population" sample or P-sample consists of the PES sample CBs drawn from the same target population, but independently conducted from the Census, for the purpose of estimating census omissions. The E-sample is the "enumeration" sample drawn from the cases already enumerated in the Census, but selected for independent re-enumeration for the purpose of estimating census erroneous inclusions. This estimate of erroneous inclusions provides a correction factor needed in the dual-system estimate of the true population.

Even though the E-sample may be separate from the P-sample, in practice, it is allowed to overlap completely with the P sample to reduce costs and improve the precision of the estimates. The E-sample then consists of the same CBs selected for the PES. A two-way match is conducted between the P-sample and the E-sample to identify both the omissions and the erroneous inclusions. The matching also produces the estimate of the "matched population" component required in the dual-system estimator³.

Table 3. Tally of a two-by-two observations⁴

	In Census	Out of Census	Total
In PES	\hat{N}_{11}	\hat{N}_{12}	\hat{N}_{1+}
Out of PES	\hat{N}_{21}	\hat{N}_{22}	\hat{N}_{2+}
Total	\hat{N}_{+1}	\hat{N}_{+2}	\hat{N}_{++}

where:

\hat{N}_{11} is an estimate of the number of people counted in both the census and the PES

\hat{N}_{12} is an estimate of the number of people counted only in the PES

\hat{N}_{21} is an estimate of the number of people counted only in the census

\hat{N}_{22} is an estimate of the number of people missed in both the census and the PES

\hat{N}_{1+} is an estimate of the total number of people counted in the PES

\hat{N}_{2+} is the total number of people counted correctly in the census (thus erroneous inclusions are factored out)

\hat{N}_{++} is the estimate of the total number of people

Dual System Estimate of the size of the total population is given by:

$$\hat{N}_{++} = \frac{\hat{N}_{+1}\hat{N}_{1+}}{\hat{N}_{11}}$$

³ Statistics South Africa, loc cit, page 43.

⁴ United Nations Statistics Division, "Post Enumeration Surveys: Operational Guidelines Technical Report" (2010), page 45.