On consistency of Gibbs-type priors

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1. Introduction

In this paper we announce results which will be extensively presented and proved in De Blasi et al. (2011) about the asymptotic posterior behaviour of Gibbs-type priors. Gibbs-type priors are a large family of discrete nonparametric priors, i.e. of nonparametric priors that select discrete distributions with probability one. In fact, a discrete nonparametric priors Q can be seen as the law of a random probability measure representable as

$$(1) \qquad \tilde{p}(\mathrm{d}x) = \sum_{i\geq 1} \tilde{p}_i \delta_{\xi_i}(\mathrm{d}x),$$

where the weights $(\tilde{p}_i)$ are random variables (r.v.s) that take value on the infinite probability simplex, while (ξ_i) form a sequence of $\mathbb{X}$-valued random locations, $\mathbb{X}$ being a complete and separable metric space equipped with the Borel σ-algebra $\mathscr{X}$. We denote by $\mathbf{P}_\mathbb{X}$ the set of probability measures on $(\mathbb{X}, \mathscr{X})$ endowed with the topology of weak convergence and with $\mathscr{P}_\mathbb{X}$ the corresponding Borel σ-field. Then Q acts as prior distribution on $(\mathbf{P}_\mathbb{X}, \mathscr{P}_\mathbb{X})$ for Bayesian nonparametric inference, that is, an exchangeable sequence (X_i) associated to $\tilde{p}$ has de Finetti measure given by Q according to

$$\mathbb{P}(X_1 \in A_1, \dots, X_n = A_n) = \int_{\mathbf{P}_\mathbb{X}} \prod_{i=1}^n p(A_i) Q(\mathrm{d}p),$$

for any $n \geq 1$ and $A_1, \dots, A_n \in \mathscr{X}$. Alternatively, it can be written in hierarchical form

$$(2) \qquad X_i|\tilde{p} \overset{i.i.d.}{\sim} \tilde{p}, \quad \tilde{p} \sim Q.$$

In studying posterior consistency of $\tilde{p}$, we adhere to the "what if" approach of Diaconis and Freedman (1986): such an approach consists in assuming the data $X_1, \dots, X_n$ to be independent and identically distributed (i.i.d.) from some "true" $P_0 \in \mathbf{P}_X$ and in verifying whether the posterior distribution of $\tilde{p}$ accumulates in any neighborhood of P_0, under a suitable topology. Here we focus on weak consistency and, then, aim at establishing whether $Q(A_\epsilon|X_1, \dots, X_n) \to 1$ P_0^∞– almost surely (a.s.) as $n \to \infty$ and for any $\epsilon > 0$, where A_ϵ denotes a weak neighborhood of P_0 and P_0^∞ is the infinite product measure driven by P_0. Given the discreteness of $\tilde{p}$, problems are expected to arise when trying to learn about a continuous P_0. This problem has been largely investigated and it is at the foundation of the research on the asymptotic properties of Bayesian nonparametric models. We briefly recall here that, following Freedman (1963), P_0 in the weak support of $\tilde{p}$ guarantees consistency only when $\mathbb{X}$ is finite. When $\mathbb{X}$ is infinite, full weak support is not enough and some additional conditions are required such as, e.g., the tail-freeness property, see Fabius (1964). In particular, the Dirichlet process satisfies the tail-freeness property and so it is consistent for any P_0 in its weak support. Interest in the study of posterior consistency for nonparametric problems was renewed by Diaconis and Freedman (1986), where it was noted one should be careful when the Dirichlet prior is used for modeling continuous distributions in semiparametric problems like inference on a location parameter. Indeed, in this kind of models the tail-freeness property does not hold true.

The convenience of investigating posterior consistency in model (2) with Gibbs-type priors lies in their fairly manageable predictive structure, see Section 2. The starting idea is that consistency in the Dirichlet case is quite apparent when looking at the predictive distribution $\mathbb{P}(X_{n+1} \in \mathrm{d}x | X_1, \ldots, X_n)$, which is a linear combination of the prior guess $P^*(\mathrm{d}x) := \mathbb{E}[\tilde{p}(\mathrm{d}x)]$ and the empirical distribution of the data. Gibbs-type priors, which contain the Dirichlet process as special case, have a similar structure and the asymptotic limit of the predictive distribution takes the following form

$$\mathbb{P}(X_{n+1} \in \mathrm{d}x | X_1, \ldots, X_n) \to \alpha P^*(\mathrm{d}x) + (1-\alpha) P_0(\mathrm{d}x), \quad P_0^\infty - \text{a.s.} \tag{3}$$

for some $\alpha \in [0,1]$. In particular, α is the asymptotic limit of the conditional probability that the $(n+1)$-th observation is different from the previous n which, using the species sampling metaphor, can be interpreted as the probability of seeing a new species. In Section 3 we state that $\alpha P^*(\mathrm{d}x) + (1-\alpha)P_0(\mathrm{d}x)$ is also the weak limit of the posterior distribution of $\tilde{p}$ so that consistency corresponds to $\alpha = 0$ while inconsistency to $\alpha > 0$. The latter is the case of the two-parameter Poisson-Dirichlet prior (another instance of Gibbs-type priors), as shown in James (2008) and Jang et al. (2010). We then focus attention to Gibbs-type priors that can be expressed as a mixture of symmetric Dirichlet distribution. We first establish a result on their weak support, then we provide sufficient condition for consistency via the asymptotic limit α in (3). In Section 4 we provide examples of Gibbs-type priors that display, for continuous P_0, the two extreme limit behaviours, $\alpha = 0$ and $\alpha = 1$. In particular, the latter accounts for the extreme situation where the Bayesian updating rule dramatically fails to learn from the data. A third example yields the whole spectrum of $\alpha \in (0,1)$ and will serve as interpretation of the two extreme cases.

We conclude this section by introducing the notation we will use in the sequel. We denote by $(x)_m$ the Pochammer symbol or rising factorial, $(x)_m = \Gamma(x+m)/\Gamma(x) = x(x+1)\cdots(x+m-1)$. For a_n and b_n sequences of real numbers, we use $a_n \sim b_n$ and $a_n \ll b_n$ for $a_n/b_n \to 1$ and $a_n/b_n \to 0$, respectively. When one or either of a_n and b_n is a random quantity, the notation $a_n \sim_{a.s.} b_n$ and $a_n \ll_{a.s.} b_n$ means that the asymptotic relation holds with probability one. We will also make use of the hypergeometric functions

$$ {}_1F_1(a; b; \lambda) = \sum_{y \geq 0} \frac{(a)_y}{(b)_y} \frac{\lambda^y}{y!} \qquad {}_2F_1(a_1, a_2; b; \lambda) = \sum_{y \geq 0} \frac{(a_1)_y (a_2)_y}{(b)_y} \frac{\lambda^y}{y!}$$

where a, a_1, a_2, b are integers, $\lambda, \eta \in \mathbb{R}$ and $|\eta| < 1$.

2. Gibbs-type priors

Gibbs-type priors (Gnedin and Pitman, 2006) belong to the class of species sampling models introduced and studied in Pitman (1996). They admit representation (1) with locations (ξ_i) i.i.d from some $P^* \in \mathbf{P}_{\mathbb{X}}$ and independent from $(\tilde{p}_i)$, so that $\mathbb{E}[\tilde{p}(\mathrm{d}x)] = P^*(\mathrm{d}x)$. If (X_i) is an exchangeable sequence of X–valued random elements from $\tilde{p}$ in (1), it is apparent that there is positive probability to detect ties in a sample of size n, for any $n \geq 2$. It is, then, natural to associate to any such sequence a partition Ψ_n of the integers $\{1, \ldots, n\}$ such that any two observations i and j belong to the same partition set if and only if $X_i = X_j$. If $C_1, \ldots, C_k$ is a possibile realizaton of Ψ_n, for some $k \in \{1, \ldots, n\}$, and $n_j = \text{card}(C_j)$, then $\sum_{j=1}^k n_j = n$. Hence the n_j's are the different frequencies with which the distinct X_i values are recorded in the sample and from the exchangeability assumption one deduces that $\mathbb{P}[\Psi_n = \{C_1, \ldots, C_k\}]$ depends only on n, k and $\{n_j : j = 1, \ldots, k\}$. Accordingly, we write

$$\mathbb{P}[\Psi_n = \{C_1, \ldots, C_k\}] = \Pi_k^{(n)}(n_1, \ldots, n_k)$$

where $\{\Pi_k^{(n)} : n \geq 1,\ 1 \leq k \leq n\}$ is a collection of symmetric functions that satisfy the addition rule

$$\Pi_k^{(n)}(n_1, \ldots, n_k) = \Pi_{k+1}^{(n+1)}(n_1, \ldots, n_k, 1) + \sum_{j=1}^{k} \Pi_k^{(n+1)}(n_1, \ldots, n_j + 1, \ldots, n_k). \tag{4}$$

$\Pi_k^{(n)}(n_1, \ldots, n_k)$ is known as the exchangeable partition probability function (EPPF) of $\tilde{p}$. The predictive distribution $\mathbb{P}(X_{n+1} \in \mathrm{d}x | X_1, \ldots, X_n)$ is recovered from the EPPF as follows. If $\boldsymbol{X}_k^{(n)}$ denotes a sample $X_1, \ldots, X_n$ featuring k distinct values, or species labels, $X_1^*, \ldots, X_k^*$ with respective frequencies $n_1, \ldots, n_k$, then

$$\mathbb{P}(X_{n+1} = \text{new} | \boldsymbol{X}_k^{(n)}) = \frac{\Pi_{k+1}^{(n+1)}(n_1, \ldots, n_k, 1)}{\Pi_k^{(n)}(n_1, \ldots, n_k)}, \quad \mathbb{P}(X_{n+1} = X_j^* | \boldsymbol{X}_k^{(n)}) = \frac{\Pi_{k+1}^{(n)}(n_1, \ldots, n_j + 1, \ldots, n_k)}{\Pi_k^{(n)}(n_1, \ldots, n_k)} \tag{5}$$

Following Gnedin and Pitman (2006), $\Pi_k^{(n)}$ is said to be of Gibbs form if there exists $\sigma \in (-\infty, 1)$ and an array of non-negative real numbers $\{V_{n,k},\ n \geq 1, k \leq n\}$ such that

$$\Pi_k^{(n)}(n_1, \ldots, n_k) = V_{n,k} \prod_{i=1}^{k} (1 - \sigma)_{n_i - 1}, \tag{6}$$

with the weights $(V_{n,k})$ satisfying the forward recursive equation

$$V_{n,k} = (n - \sigma k) V_{n+1,k} + V_{n+1,k+1}. \tag{7}$$

The predictive rule induced by (6) is such that the probability of observing a new species at the $(n+1)$-th observation, given $\boldsymbol{X}_k^{(n)}$, depends only on n and k but not on the frequencies n_j, i.e.

$$\mathbb{P}(X_{n+1} = \text{new} | \boldsymbol{X}_k^{(n)}) = \frac{V_{n+1,k+1}}{V_{n,k}}, \tag{8}$$

whereas

$$\mathbb{P}(X_{n+1} = X_j^* | \boldsymbol{X}_k^{(n)}) = \frac{V_{n+1,k}}{V_{n,k}} (n_j - \sigma).$$

Upon definition of the the pseudo-empirical distribution $\tilde{P}_{n,k}(\mathrm{d}x) = \frac{1}{n - \sigma k} \sum_{i=1}^k (n_i - \sigma) \delta_{X_i^*}(\mathrm{d}x)$ the predictive distribution takes the form of a linear combination of P^* and $\tilde{P}_{n,k}$,

$$\mathbb{P}(X_{n+1} \in \mathrm{d}x | \boldsymbol{X}_k^{(n)}) = \frac{V_{n+1,k+1}}{V_{n,k}} P^*(\mathrm{d}x) + \left(1 - \frac{V_{n+1,k+1}}{V_{n,k}}\right) \tilde{P}_{n,k}(\mathrm{d}x)$$

where $1 - (V_{n+1,k+1}/V_{n,k}) = V_{n+1,k}/V_{n,k}(n - \sigma k)$ is the probability of having a tie. Gibbs-type priors define a broad family of random probability measures including, as a special case, the two-parameter Poisson-Dirichlet process (Perman et al., 1992), which corresponds to the following solution of (7):

$$V_{n,k} = \frac{\prod_{j=1}^{k-1} (\theta + j\sigma)}{(\theta + 1)_{n+1}} \tag{9}$$

where the possible value of the parameters (σ, θ) are $\sigma \in [0,1)$ and $\theta > -\sigma$ or $\sigma \in (-\infty, 0)$ and $\theta = x|\sigma|$ for some $x \in \mathbb{N}$. For $\sigma > 0$, it is well-known that the random probability masses $(\tilde{p}_i)$ in (1) admit a stick-breaking representation. Indeed, if (W_i) is a sequence of independent random variables, with $W_i \sim$beta$(\theta + i\sigma, 1-\sigma)$, then $\tilde{p}_1 = W_1$ and $\tilde{p}_j = W_i \prod_{j=1}^{i-1}(1-W_j)$. The Dirichlet process arises when $\sigma = 0$ and, in this case, the probability that X_{n+1} reveals a new species, given $\boldsymbol{X}_k^{(n)}$, does not depend on k: such a feature actually characterizes the Dirichlet process. Finally, parameters $\sigma < 0$ and $\theta = x|\sigma|$, for some positive integer x, corresponds to an x–variate symmetric Dirichlet distribution on the simplex with parameter vector $(|\sigma|, \ldots, |\sigma|)$.

3. Consistency of Gibbs-type priors

An important goal of our investigation concerns the determination of the asymptotic behaviour of $V_{n+1,k+1}/V_{n,k}$ when assuming that the observations (X_i) are i.i.d. from P_0. Obviously, the sample $X_1, \ldots, X_n$ influences the ratio in (8) through k: for this reason we shall use the notation k_n henceforth. One identifies two different behaviour of k_n: when P_0 is discrete, $k_n \ll_{a.s.} n$, see Gnedin et al. (2007), while P_0 continuous yields $k_n = n$ a.s. Henceforth we will make the following assumption:

$$\text{(A)} \qquad \frac{V_{n+1,k_n+1}}{V_{n,k_n}} \to \alpha \text{ as } n \to \infty \quad P_0^\infty - \text{a.s.}$$

for some constant $\alpha \in [0,1]$. Theorem 1 below states that (A) means that the posterior for sufficiently large n tends to concentrate around the predictive distribution. This is achieved by using a bound for the posterior variance of $\tilde{p}(A)$, for any A in $\mathscr{X}$, in such a way that $\mathbb{V}ar[\tilde{p}(A)|\mathbf{X}_k^{(n)}] \to 0$ P_0^∞–a.s is implied, see De Blasi et al. (2011). At the same time, (A) determines the asymptotic limit of the predictive distribution. James (2008) adopted a similar approach in investigating consistency of the two parameter Poisson-Dirichlet process.

Theorem 1 *Let $\tilde{p}$ be a Gibbs-type prior with continuous prior guess P^* and let the support of P^* coincide with $\mathbb{X}$. If (X_i) are i.i.d. from P_0 such that* (A) *is in force for some $\alpha \in [0,1]$, then, the posterior converges weakly to a point mass at $\alpha P^*(\cdot) + (1-\alpha)P_0(\cdot)$.*

According to Theorem 1, the posterior is consistent if and only if $\alpha = 0$ in (A). It reduces to Proposition 1 of James (2008) when $\tilde{p}$ is two-parameter model with $0 \le \sigma < 1$. In fact, from (9) one has $V_{n+1,k+1}/V_{n,k} = (\theta + \sigma k)/(n+\theta)$. Hence, when P_0 is discrete and $k_n \ll_{a.s.} n$, we have $\alpha = 0$, implying consistency. When P_0 is continuous and $k_n = n$ a.s., we have $\alpha = \sigma$, hence inconsistency, unless $\sigma = 0$, which corresponds to the Dirichlet case. See also Theorem 1 in Jang et al. (2010).

The practical implication of Theorem 1 is that consistency (or inconsistency) of Gibbs-type priors boils down to study the asymptotic behavior of $V_{n+1,k_n+1}/V_{n,k_n}$ along the random path (n, k_n), $n \ge 1$ and $1 \le k_n \le n$, induced by the i.i.d. sampling from P_0. We will focus next to the case of $\sigma < 0$ since in this case $V_{n,k}$ has a nice mathematical form which allows to establish a quite general result. In fact, the Gibbs-type EPPF with negative σ is a mixture of the corresponding two-parameter model with $\sigma \in (-\infty, 0)$, $\theta = |\sigma|x$ and $x \in \mathbb{N}$, that is

$$V_{n,k} = \sum_{x \ge k} V_{n,k}^{\sigma,x}\pi(x), \quad V_{n,k}^{\sigma,x} := \frac{\prod_{j=1}^{k-1}(|\sigma|x + j\sigma)}{(|\sigma|x+1)_{n-1}}, \quad k \le x$$

cfr. (9), where $\pi(x)$ is a probability measure on the number of species, hence supported on the set of positive integers $\mathbb{N}$. Consequently, the sequence of weights $(\tilde{p}_i)$ is identified by the mixture model

$$\begin{aligned} \tilde{p}_1, \ldots, \tilde{p}_K &\sim \text{Dirichlet}(|\sigma|, \ldots, |\sigma|) \\ K &\sim \pi(\cdot) \end{aligned}$$

Such a Gibbs-type prior has full weak support as stated in the following proposition.

Proposition 1 *Let Q be the law of a Gibbs-type prior with $\sigma < 0$ and mixing measure π. If $\pi(x) > 0$ for any $x \in \mathbb{N}$ and the support of the prior guess P^* of $\tilde{p}$ coincides with $\mathbb{X}$, then the weak support of Q coincides with $\mathbf{P}_{\mathbb{X}}$.*

We proceed now to the investigation of condition (A), that is the study of the asymptotic limit of $V_{n+1,k_n+1}/V_{n,k_n}$. Theorem 2 below gives sufficient conditions for consistency in terms of the tail of the mixing $\pi(x)$. As in Proposition 1, we assume that $\pi(x) > 0$ for any $x \in \mathbb{N}$.

Theorem 2 *Let $\tilde{p}$ be a Gibbs-type prior with parameter $\sigma < 0$, mixing measure π, and continuous prior guess P^* whose support coincides with $\mathbb{X}$. Then*

(i) the posterior is consistent for any discrete P_0 if

(B) $$\pi(x+1)/\pi(x) \leq 1, \quad \textit{for } x \textit{ sufficiently large};$$

(ii) the posterior is consistent for any continous P_0 if (B) *is strengthened to*

(B') $$\frac{\pi(x+1)}{\pi(x)} \leq \frac{M}{x}, \quad \textit{for some } M < \infty \textit{ and } x \textit{ sufficiently large.}$$

Condition (B) is a very mild assumption on the regularity of the tail of the mixing $\pi(x)$: it requires $\pi(x)$ to be monotonically decreasing. Condition (B') requires instead that the tail is sufficiently light.

4. Illustration

In this section we consider three examples of Gibbs-type priors with $\sigma = -1$ determined by different mixing $\pi(x)$ on the number of species. In all of them we have consistency for discrete P_0 as implied by part (i) of Theorem 2 since $\pi(x)$ satisfies (B). Different results are obtained for continuous P_0. In the first example, the mixing $\pi(x)$ has heavy tail and we have $\alpha = 1$ so that the posterior concentrates around the prior guess P^*. The second example, where the mixing $\pi(x)$ has sufficiently light tail, we get consistency. In the third example α takes values over the whole unit according to a parameter that determines the tail of $\pi(x)$. It indicates that the lighter the tail, the lighter the limiting mass assigned to the prior guess.

Example 1 We consider a family of Gibbs-type prior with $\sigma = -1$ recently introduced by Gnedin (2010). It has mixing given by

$$\pi(x) = \frac{\gamma(1-\gamma)_{x-1}}{x!}, \quad \gamma \in (0,1).$$

Such $\pi(x)$ arises in discrete renewal theory, see Feller (1971, chapter XII), and in connection with the two-parameter Poisson-Dirichlet prior with $0 < \sigma < 1$, see Pitman (1997). It has heavy tail and no finite moments, as it can be seen by looking at its probability generating function, see De Blasi at al. (2011) for details. Note that (B') is not satisfied, since $\pi(x+1)/\pi(x) = (x-\gamma)/(x+1)$ is increasing in n and converges to 1 from below. The asymptotic limit α of the probability of observing a new species $V_{n+1,k_n+1}/V_{n,k_n}$ is easily derived since $V_{n,k}$ has explicit form:

$$V_{n,k} = \frac{(k-1)!(1-\gamma)_{k-1}(\gamma)_{n-k}}{(n-1)!(1+\gamma)_{n-1}}$$

Hence

(10) $$\frac{V_{n+1,k_n+1}}{V_{n,k_n}} = \frac{k_n(k_n-\gamma)}{n(\gamma+n)}$$

which goes to 1 as $n \to \infty$ when P_0 continuous and $k = n$ a.s.. So $\alpha = 1$ and the posterior concentrates around the prior guess P^*. From (10) it is also immediate to check that $\alpha = 0$ when P_0 is discrete and $k \ll_{a.s.} n$, hence a confirmation of part (i) of Theorem 2.

Example 2 We now consider a mixing $\pi(x)$ with light tail:

$$\pi(x) = \frac{\mathrm{e}^{-\lambda}}{1-\mathrm{e}^{-\lambda}} \frac{\lambda^x}{x!}, \quad \lambda > 0$$

that is the Poisson distribution rescaled on the positive integers. Condition (B') is satisfied since $\pi(x+1)/\pi(x) = \lambda/(x+1)$ with $M = \lambda$. From part (ii) of Theorem 2, the posterior is consistent also when P_0 is continuous, that is $\alpha = 0$. The same conclusion can be drawn by direct calculation since $V_{n,k}$ can be expressed in terms of confluent hypergeometric function:

$$V_{n,k} = \pi(k) V_{n,k}^{-1,k} {}_1F_1(k; k+n; \lambda)$$

Recalling from De Blasi e al. (2011), when $k_n = n$, one has

$$\frac{V_{n+1,k_n+1}}{V_{n,k_n}} \sim \frac{\lambda}{2(2n+1)} \to \alpha = 0, \quad \text{as } n \to \infty.$$

Hence, when P_0 is continuous, the probability of observing a new species converges to $\alpha = 0$, in accordance with part (ii) of Theorem 2.

Example 3 Consider now the case of geometric mixing:

$$\pi(x) = (1-\eta)\eta^{x-1}, \quad \eta \in (0,1).$$

Note that $\pi(x+1)/\pi(x) = \eta$ so that (B') is not satisfied. Hence one needs to work out the limit of $V_{n+1,k_n+1}/V_{n,k_n}$ when P_0 is continuous and $k_n = n$ a.s.. It turns out that $V_{n,k}$ can be expressed in terms of the hypergeometric function ${}_2F_1$:

$$V_{n,k} = \pi(k) V_{n,k}^{-1,k} {}_2F_1(k, k+1; k+n; \eta)$$

and, recalling from De Blasi et al. (2011), one has that, when $k_n = n$,

$$\frac{V_{n+1,k_n+1}}{V_{n,k_n}} \to \alpha = \frac{2-\eta-2\sqrt{1-\eta}}{\eta} \in [0,1]$$

Such α spans the whole unit interval according to the value of η, so that we have the whole spectrum of weak limit $\alpha P^*(\cdot) + (1-\alpha)P_0(\cdot)$, see Theorem 1. In particular, α is increasing in η, so the larger η, the heavier the limiting mass assigned to the prior guess. Small values of η are consistent with what happens for the Poisson since small η corresponds to light tail. Conversely, large values of η are consistent with what happens in Example 1 since large η corresponds to heavy tail.

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ABSTRACT (RÉSUMÉ)

Most of the currently used discrete nonparametric priors are, with the exception of the Dirichlet process, inconsistent when used to model directly continuous data. On the other hand, when specified as basic building blocks within hierarchical mixture models, they generally lead to consistent density estimation. In this paper we announce several asymptotic results for a large class of discrete nonparametric priors, namely the class of Gibbs-type priors, which will be extensively presented and proved in De Blasi et al. (2011). Specifically, we provide sufficient conditions for consistency and present two examples within this class which exhibit completely opposite asymptotic posterior behaviours when the "true" distribution is continuous.