Research and Recent Developments in Probability Education: an Interactive and Electronic Approach

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1. Background

This paper builds on the current research efforts in the teaching of probability across the world, as discussed at the last ICME in Mexico in 2008 and CERME 2011. The main aim of this paper is to discuss themes in probability education research and the influence of new technology in how research is presented and how this may change even the nature of research so that this revolution will encourage more collaborative and interactive approaches across countries, languages and cultures.

Whether probability education needs to be seen as discrete and separate from statistics has been an ongoing debate for many decades. Nowadays, statistics seems to be dominant in school education and data handling has been a key theme as part of the movement of mathematics for all. Conversely probability is thought to be harder and less relevant. Nevertheless, probability is an important discipline in its own right, and does contain the key underpinning concepts to understand and use data sensibly. Revived interest in probability comes from very recent considerations to integrate some concepts of risk analysis into teaching. Mentions of the crisis with Lehman brothers and the possible epidemic of swine flu evoke the rationale for this assertion. This paper describes current research in probability and also focuses on the electronic aspects which will support future research internationally.

Despite the fact that a multitude of new technologies is available now in the era of information technology and multi-media is spreading to all corners of life, publication in research has hardly changed. Textbooks for study are changing gradually, a few hypertexts make use of the possibilities of new media but for research publications it seems that times have not changed yet. Borovcnik (2007) has analyzed the consequences of new technologies on applications and on educational endeavours; more endeavour is needed from the research community to improve its communication. Borovcnik & Kapadia (2009) have taken up the challenge of innovative publication (including multi-media and more) in the age of web 2.0. Initially we focus on the main streams of research in educational probability by classifying the international endeavours to obtain new insights on the teaching of probability. These are from ICME 11 or CERME 7.

2. Overarching Themes in Probability Education Research

Probability and statistics have been part of school mathematics for less than 40 years and complement the traditional topics of arithmetic, algebra and geometry. Statistics is part of the school curriculum across the world but ideas of probability may only be introduced for older pupils. Application-oriented statistics is undisputed in its relevance, but the place of probability is more ambivalent. Reduction of probability to the classical conception, mainly based on combinatorics, and its perception as a solely mathematical discipline with its connection to higher mathematics, are sometimes used as arguments to abandon it in favour of the statistics part. However, there are several key reasons for a strong role for probability within mathematics curricula:

1. Misconceptions on probability affect people‟s decisions in important situations, such as medical tests, jury verdicts, and investments.
2. Probability is essential to understand any inferential procedure of statistics.
3. Probability offers a tool for modelling and "creating" reality, such as in physics.
4. The concepts of risk (not only in financial markets) and reliability are closely related to and dependent upon probability.
5. Probability is an interesting subject in its own right and worthy of study.

The challenge is to teach probability in order to enable students to understand and apply it, by creating approaches that are both accessible and motivating. Both, the frequentist and subjectivist views of probability, and connections of probability to practical applications should be taken into account. Simulation is one such strategy, as is visualization of abstract concepts.

In accordance with endeavours to focus on modelling and applications (the main stream in teaching in reaction and critique to the „New Math‟), through the curricula one should search for connections of probability to practical applications. With respect to didactical aids, visualization of abstract concepts is important in teaching; for probability, it has also become standard to visualize the consequences of models by the simulation technique. There are more possibilities to facilitate learning. The use of technology helps to reduce the technical calculations and focus the learner on the concepts instead. The world of personal attitudes and intuitions is another source for success or failure of teaching. This might be true for other disciplines of mathematics; however, it is especially important for probability, as there seem to be strong emotions connected to the concepts and random situations, which decide also whether students *accept or ignore* what they learn.

The main themes in current probability education research, which nevertheless do overlap are: Pre- and Misconceptions; Conditional Probability and Bayes‟ Theorem; Impact of Technology; Fundamental Ideas and Risk.

2.1 Pre- and Misconceptions

The rules of probability might appear to be simple. However, they all involve probabilities and as such a rather theoretical concept. What does a probability of ½ or 0.2 really mean? The individual‟s world is full of diverging private conceptions connected to probability statements. Even if a student has adopted a frequentist interpretation and even if the probabilities are moderate (i. e., in a range of 0.1 to 0.9) and even if there are no high losses or wins at stake (which also might involve intuitions about their utility), there remains much to clarify through teaching including the law of large numbers (and the many inadequate conceptions to it), or including the task of using such probabilities for decisions. Single trials, the overall outcome, and connections between the two are also important. Abstracting a general law from various single outcomes (which also involves mathematical concepts) is not too easy to understand, nor applying such a general law again to single outcomes etc.

Technology might help to build stable intuitions but the private conceptions are so unstable that using technology to simulate a random experiment repeatedly and analyze it by inspecting the results from various angles does not always help. To perform an experiment with "real world devices" might be very different for younger students from performing such experiments on the computer that they would develop completely different attitudes and conceptions thereby. As is commonly known, younger students often find it difficult to accept that to throw two dice simultaneously amounts to the same experiment as to throw a single die two times.

Ben-Zvi et al (2011) analyzed children‟s reasoning about sampling when making informal statistical inferences using interviews with two articulate academically successful boys from a design experiment in

Israeli Grade 5 (age 11) classrooms. Initially, the boys‟ emergent reasoning between deterministic and relativistic conclusions, but they came to reason in more sophisticated ways over time, as the sample sizes grow from 8 to 30 to 90 to 270. The boys qualified their claims as only holding for their limited data and were later surprised that the larger data set confirmed many of their previous conclusions. The research confirmed that an inquiry-based learning environment, with suitable tasks and tools as well as teacher scaffolds, supports the development of informal inferential reasoning.

Chernoff (2011) undertook research on prospective teachers‟ understanding of probability. He used two multinomial sequences relating to the answer key for a 10-question multiple-choice test. The context leads the teachers to rate one sequence as more likely because of the context, as confirmed in their written explanations. Chernoff asserts that the results show that context has a significant influence on probability judgments, a fact which needs to be bore in mind by researchers when analysing results.

The empirical studies by F. Chiesi and C. Primi (Italy) and L. Zapata (Colombia/USA) deal with heuristics. The Italian study dealt with the *development* of “negative” and “positive recency” with age. They compare 9, 11, and 25 years olds in order to imitate a longitudinal survey (also available on-line and so can be replicated elsewhere).

> *From a bag with blue and green marbles a marble is drawn repeatedly with replacement. The result “all marbles of the same colour” is presented (the marbles are not actually drawn). The numbers of both colours are known – they are varied from equal numbers to a strong bias to either of the colours.*

According to the “negative recency”, people predict a change: with 4 blue they would predict a green one for the fifth draw. With the “positive recency”, they predict that colour, which continues the series “observed”. How frequently are these heuristics used, and are they independent of the composition of the colours in the bag? Interestingly, the study shows an increase of the normative (correct) solution first (from age 9 to 11), but then this drops down (age 25). To a similar extent, “negative recency” decreases (from age 9 to 11) first and then increases again (age 25). With the “positive recency” there is a decrease (9 to 11) and it remains amongst adults (25) at this level. There have to be more in-depth investigations to clarify what is happening, and whether such a “development” can be confirmed (of course it is not truly longitudinal).

Primi & Chiesi (2011) also used a new model developed by Stanovich and colleagues about the ways that knowing the relevant normative rules and procedures (called “mindware”) influences normative performance in solving problems. They started with the gambler‟s fallacy relating to predictions in a probability experiment about drawing balls from a bag. They allowed for cognitive ability by using carefully derived measures. Specifically, they examined the interactions between relevant knowledge and cognitive ability in gambler fallacy tasks. They found that mindware does play an important role in probabilistic reasoning independent of age. Moreover, once cognitive ability has been taken into account, probabilistic reasoning continues to rely strongly on knowledge about normative rules. When students possess relevant normative knowledge, primary and college students perform equally well. Yet, when they do not possess the knowledge, their performance is equally poor, ignoring the base-rates and the independence notion, in committing the gambler fallacy and relying on intuition and heuristics. This shows the importance of acquiring the relevant knowledge (acquired through education) on reasoning ability.

It may be that probability is much more prone to such difficulties than other topics in mathematics. This is confirmed by V. Kataoka (Brazil) who ran a series of workshops of in-service education. One special experiment used in the workshop illustrates the importance of suitable models and data sampled by randomness (when do you really have data from random samples?). Suppose we break a stick randomly into three pieces. Afterwards, we try to form a triangle of the three pieces. The success rate to form triangles can be determined - try it with spaghetti.

In practice, success rates of over 50% are not rare. In contrast to it, there are (at least) two models for randomly breaking the stick (with 25% and 19% success rates). The obvious discrepancy between the theory and the model lets us gradually start to doubt whether we can break the stick truly randomly into 3 parts. As a conclusion, relative frequencies might sometimes be of no value to estimate an unknown probability. This enriches the usual discussion about the convergence of relative frequencies by focusing on the underlying assumption of randomness of the data. Analogous examples are available (such as Buffon‟s needle) but perhaps less emotionally laden than spaghetti.

2.2 Conditional Probability and Bayes' Theorem

Conditional probability and Bayes‟ theorem are important ingredients and should not be left out of any course in probability at school and at university, including for non-mathematical students. The concepts overlap with many diverging private conceptions, which are also causally interwoven. These concepts also stand at the "crossroads" between the two different conceptions of theories of probability, the objectivist conception with mainly the frequentist interpretation and the subjectivist conception with probability as degree of confidence. For such reasons, to learn only the mathematics of these concepts does not suffice to understand and apply them adequately.

K. Rolka (Germany, working jointly with S. Prediger) presented a study of 12 year olds playing a game of fortune with tokens moved forward on a playing board by the result of a die. The icosahedron used had more red sides than sides of any other colour whence it favoured the red token. The discussion amongst the children reveals how they interact and argue fiercely in favour of their strategy. Finally they *jointly* agree on an optimum strategy. Views such as "there are more red sides on the die" and "the red token wins more often" were advocated likewise. The *common* struggle for a strategy seems to generate a better understanding (or at least a better acceptance) of the value of their strategy – they seem to be much more aware of the chance that a token of a different colour could still win as a result of their discussion; much more than they were aware by simply playing or *observing* the game. The social situation of the class with the children interacting in their discussion is a feature of the investigation – the situation is thought also to be exemplary for later teaching. The remark about understand and accept what one learns is not arbitrary. In probability, there seems to be something more: even if one understands a concept, a child might not be willing to apply it. In school, we perform the „glass bead game‟ of Hesse, privately we solve it by our intuitions. The social construction, the joint struggle for a solution might be a teaching strategy to counter such a phenomenon.

Many different types of errors have already been investigated in isolation. According to C. Batanero and C. Diaz (Spain), however, there is neither a study investigating connections between various types of misconceptions, nor an analysis whether misconceptions are related to mathematical knowledge, i.e., whether they decrease with better achievement in mathematics. Consequently, they have developed a test with (mainly familiar) items, and administer it to university students. Data are analyzed by means of factor analysis. They describe some phenomena, which remain even with higher mathematics education, but in general a significant decrease in misconceptions is found with a higher level of mathematics. For interrelations between several misconceptions, the result is less optimistic as these misconceptions seem to be quite isolated with not many relations in between. As a consequence of this investigation, endeavour in mathematics education in probability has to be fostered while the types of misconceptions still have to be singly put to the fore in teaching again and again in order to facilitate students‟ understanding. The test is now available electronically (in English and the Spanish original) and so can be used elsewhere to gain more insight across cultures etc.

Contreras et al (2011) assessed the common content knowledge (CCK) and specialized content knowledge (SCK) of probability in a group of prospective primary school teachers in Spain, with a task to find probabilities from a 2-way table. Only two-thirds of the group was able to calculate a simple probability and less than half was able to find conditional and compound probabilities. This showed a lack of common content knowledge one might expect from ideas learnt in school. The teachers also had limited specialised content knowledge of the underlying ideas in the problems presented. They note that a future teacher should master both the concept and the language used in teaching. They state that the results are a cause for concern and suggest the need to reform and improve the probability education these future teachers receive.

P. Huerta (Spain) criticizes a serious flaw of some existing research on conditional probability problems, which does not take the structure of the posed problems into account. He describes a mathematical structure of "ternary problems" and classifies 20 different types of problems with conditional probabilities of which only *one subclass* (and from it mainly one type of task) has been used in existing research. From this insight, it becomes questionable how to generalize from the research results obtained to other kinds of problems with conditional probabilities. He uses graph theoretic methods (ternary graphs with edges connecting three nodes each) to describe the world of all conditional probability problems. Each single problem amounts to a subgraph of the full graph. Solving the problem is mapped by a successive extension of this graph until the target point is connected to it. Thus, he can visualize the steps of solution and the complexity or difficulty of a special problem at hand. By this structural analysis, Huerta develops a plan for future empirical research to cover all types of conditional probability problems to enhance the insights, which might be gained. In later stages of research he plans to extend the research from mere analysis of a subject"s behaviour to classroom analysis in order to evaluate teaching interventions for their relative success. His ideas are complex but the addition of visual representations, which can be accessed via hyperlinks helps enormously to understand the potential of his approach.

Huerta et al (2011) have also developed a very detailed approach to assessing the difficulty of problems in conditional probability. They have devised special terminology: ADP – appreciated problem difficulty (as assessed by a student"s response); PD -problem difficulty; PSD- problem solution difficulty; CDS – correct description of the solution. They try to measure each of these objectively using students" written answers. A scale of 0 (easy) to 100 (impossible) is derived. Subsequently he devises isomorphic problems in a range of contexts such as social or health. The problems are graded by level of difficulty using a systematic approach. The questions have been administered to two groups of students: one group is aged 15-16; the other group is training to become secondary mathematics teachers. The results show very low percentages of correct solutions. They suggest that a teaching model based on solving problems belonging to distinct groups, organized sequentially, and explored through varied contexts. This could potentially be a very good way to improve students"competence in solving conditional probability problems.

There are pitfalls in the interpretation of results from statistical tests or from confidence intervals. These originate from the reduction of the interpretation of probability to situations, which may be repeated independently in the same manner. On this issue there has been a vigorous debate not only in the foundations of statistics but also in the didactical community. Ö. Vancsó (Hungary) has developed a parallel course in classical *and* Bayesian statistics. He believes that it is a false dichotomy, to teach *either* classical statistics *or* Bayesian statistics, as both offer a consistent theory of probability. He has tried and refined his ideas in several cycles in teacher pre-service education and reported his positive experiences: "Now I have really understood what is meant by confidence intervals" one of his students exclaims.

An interesting extra-curricular activity was explored by H. Trevethan (Mexico) who described a project in the context of a science fair. A pair of students worked on a project to present a game of chance. There were several advantages. These included the autonomous activity of the students, their own responsibility, presenting in public etc. The game was "Shut the box", which is certainly open to varied stochastic strategies as different dice can be chosen to play the game. This authentic (and not artificial) transfer of responsibility could well be taken up more often in teaching in class. Mathematically, conditional probabilities and Bayes"theorem are the key concepts to develop winning strategies.

2.3 Impact of Technology

Technology can be viewed in at least two very distinct ways. In one aspect the media is used such as Powerpoint or interactive use of computers by students. The other aspect relates to the software tools used, such as Excel, Fathom etc. Some software is generic (e. g., Excel) and some software is designed specifically for probability such as ChanceMaker. In practice there is more software relating to statistics, though probability software is growing.

Inzunsa (Mexico) used Fathom to help the students understand connections between empirical and theoretical distributions by simulations; a crucial point thereby is a solid perception of small tail probabilities, which are used in statistical inference for evaluating significance of observed results. Peard (Australia) has developed a whole course on games of fortunes with EXCEL. An interesting side argument with his approach is to consider games of fortune not as abstract, idle situations but – in the face of the huge business behind such games – as *applications*. Probability supported by calculations and simulations in EXCEL may contribute to a clearer perception of such games for the young students.

2.4 Risk and Fundamental Ideas

The fundamental ideas of probability include random variables, distribution, expectation, and relative frequencies, as well as the central limit theorem. Games of chance have been partially discredited by their closeness to combinatorics (which is not always easy to understand) and by their artificiality (we want to teach real applications to our students). However, games of chance have spread widely, such as lotteries, and developed to become an important business sector, which is still growing. M. Borovcnik (Austria) has studied some peculiarities of stochastic thinking, which make it so different from other approaches. He notes that:

- There is no direct control of success with probabilities – the rarest event may occur and "destroy" the best strategy.
- Interference with causal re-interpretations may lead a person completely astray.
- Our criteria in uncertain situations may stem from "elsewhere" and may be laden with emotions – probability and divination have a common source in ancient Greece.

With these features of stochastic thinking in mind, paradoxes like the stabilizing of relative frequencies, even though new events have full-fledged variability, may not seem special. One difficulty lies in a primitive attribution of an ontological character of probability to situations. Probability does not exist – it is only one of many views to reflect on phenomena of the real world.

R. Kapadia (England) studied tasks from the national tests and concludes from the poor achievement of students therein that teaching compared to 20 years behind has not really improved substantially. This may be rooted back to recent trends as statistics, mainly that simple data handling is favoured at the cost of probability in the curricula. And, as far it concerns probability, teaching still focuses too much and narrowly on equal likelihood and experimental probability, there is not enough work on misconceptions, and risk as a concept related to probability is hardly discussed. Clearly, if people judge probabilities, they will have a strong bias towards "equal probabilities", especially when they are (or feel) confronted with two possibilities. The fundamental idea of judging probabilities and risks subjectively – coloured and supported by qualitative and objective information – has still not found a sustainable form of teaching. The questions used are available for readers to consider for themselves.

Nikiforidou and Pange (2011) worked with pre-school children on two risk-taking tasks. In one, children made a gain by taking a risk, in the other they minimised a loss by taking a risk: in both cases, the chance of doing worse with a risk was 33%. In both tasks, children were willing to take a risky option and they did this more often in the latter task to avoid losses. This experiment demonstrated that even young children have an awareness of risk and prefer to avoid losses by taking a risk. This complies with the preference shift phenomenon. Under this perspective, choices involving gains are risk averse and choices involving losses are risk seeking. One of the basic features of decision under risk and uncertainty is that losses loom larger than gains. Risk seeking is prevalent when people must choose between a sure loss and a substantial probability of a larger loss and alternatively, people often prefer a small probability of winning a large prize over the expected value of that prospect. These findings indicate that children posses a limited understanding of probabilistic notions.

Pratt et al (2011) explored the prevalence of the priority heuristic in an experimental situation set up by a discussion between a mathematics and a science teacher. They noted that the two teachers placed highest priority on losses (rather than probabilities) and that they decided according to the lesser of the minimum losses that would be incurred by two possible decisions. They also note that the teachers‟ use of the priority heuristic is far from robust and influenced by a simple intervention. They assert that the evidence that people avoid trade-offs and apply heuristics such as the priority heuristic is compelling. This presents the pedagogic challenge to find ways to support thinking that engages more explicitly with trade-offs by facilitating the co-ordination of the various dimensions of risk. They envisage new tools that can list and order hazards by size of risk, that consolidate harm, likelihood as well as ethical and moral dimensions, using features such as a concept-mapping tool. They conjecture that such tools might provide an educational intervention that would enable teachers, and later students, to coordinate the dimensions of risk into a single construct with the promise that they might, under certain circumstances, use thinking about trade-offs rather than strategies, such as the priority heuristic.

J. Watson (Australia) also focuses on probabilities linked to risks. Such issues often involve major consequences, utilities and small probabilities. This combines potential (fictional) adverse consequences (which have not occurred or are very rare) with misleading intuitions hindering rational thought. M. Borovcnik (Austria) develops fundamental ideas by examples; *thinking probabilistically* goes beyond thinking in mathematical models; *weighing the evidence* depends heavily on the role, which one plays in a situation; *evaluating relative frequencies* requires overcoming the paradox of stabilizing and yet at the same time fluctuating.

3. Teaching Implications

Few of these research studies point to clear implications for the teaching of probability, though some ideas have been included above. The research of Kahneman and Tversky was initially undertaken within their discipline of cognitive psychology though the Nobel Prize for their work was awarded in economics. In fact the work relates most closely to the mathematical discipline of probability. In the 1970s, when the research was undertaken, probability was not taught in schools and featured only briefly in statistics courses for non-mathematicians in universities. Yet though the problems were identified almost four decades ago, at least some of the problems and difficulties seem to remain. The messages from the Kahneman- Tversky research for education are not clear-cut. The misconceptions and erroneous reasoning they highlighted does need to be addressed. Certainly, the curricular situation has changed dramatically across the world: many countries have included the teaching of probability and statistics, sometimes even in primary school and almost invariably at secondary school. The topics also feature at university level, including for prospective teachers.

The continuing research in this area does document improvements in probabilistic reasoning with the teaching of probability in school. In some cases, difficulties persist, but a hopeful paper in this respect is the one by Primi & Chiesi (2011), reported above. It shows that fallacies and paradoxes can be addressed through systematic teaching, which continues to support students‟ probabilistic reasoning at university. The key is that sufficient emphasis needs to be put on developing the ideas, including time for discussion and collaboration to confront intuitive pre-conceptions.

Nevertheless, there has been a move during this century to reduce the emphasis on probability and concentrate more on statistics, especially data-handling. In some countries, probability has also been removed as a topic in primary school. This research shows that the approach is misguided. This is especially true now - with an increased perception that assessment of risk – which ultimately has to be based on probability – has become increasingly important.

4. Electronic Publishing: Perspectives for the Future

We go on to discuss how electronic publishing, such as undertaken in IEJME 4(3) by Borovcnik and Kapadia (2009) affects the research and how it might pave the way for future research, especially for younger researchers. For it has become clear that the nature of the research changed in the process, as well as the way that ideas can be developed in the future. We describe here some key elements on the way technology has been used in IEJME 4(3). These include the use of several languages, and interactive features. Significantly, every paper published in this special issue of IEJME contains some electronic aspects, which are not available in a traditional format.

One key element is that the abstract of a paper is translated into at least two other languages. The availability of abstracts in three languages helps readers across the world. English is accepted as the main language of communication in much of the academic world. This is a second language for most people and can be a significant constraint. It is hoped that those for whom English is a second or even third language can be engaged by this approach. It can be off-putting to begin to read a research paper when one is unsure of the thrust of the paper; in IEJME 4(3) the abstract is accurately presented in Spanish or German, using the skills of a native speaker who is also familiar with the research ideas. This combination will, we believe, encourage much more international readership for the electronic papers in IEJME.

Another possibility which has been exploited is to offer background material to the underlying research. For readers who are especially interested, they can analyze such material themselves. This will give a deeper insight into the research approach taken in the paper. Even the availability of direct links to a reference in the paper enables immediate access to ideas which are quoted. Sometimes this can be a book, sometimes it will be a paper in another journal, sometimes it is material on the internet. Occasionally there is a link to related software, or even to games, as referred to in some papers. The point is to separate the justification and referencing of ideas, as well as referring to literature beyond the normal citations. A reader may also explore why an author cites a particular source. One might also wonder how a source is embedded in a wider spectrum of research, as well as alternative approaches. This enables a balanced evaluation of how and why to cite, which is valuable for researchers to explore varied approaches. The paper by Abrahamson is an example where there is a companion paper. Moreover, wherever possible, hyperlinks are included within references; such has been done with this paper. This means that a reader can access a reference immediately rather than ordering it through a library and waiting for its arrival.

There is also extra-material such as video-recordings or taped interviews; this is a much richer source than simple transcripts which cannot convey tone or visual expressions and interactions. These form the basis of some of the research and enables a reader to draw his/her own conclusions, as well as understand the author‟s conclusions more fully. Interactive ideas are no longer only formulated but are explained by visualizing, as well as by graphs and animation. Communication in research comes hereby closer to "real communication" and not just describing results. Like a good hyper-textbook for students, this is a hyper-textbook for researchers. For example, the test used by Batanero is available, as well as transcripts of interviews by Watson.

Some papers include Excel spreadsheets. These are included in an interactive format to allow the reader to make explorations. Test results are also included interactively; for example, the table of results can be manipulated so that the results can be in an order chosen by the reader, such as according to facility rate, or with certain items grouped together. Colour has also been included so that results can be analysed with more clarity.

These aspects will be particularly valuable to researchers who could also see the material in the background as specimen for their own future work. One cautionary remark is also in order: electronic publishing in this way takes much more time and effort, both for the author and also for the editor. Yet every paper includes some minimum aspects of electronic publication: abstracts in three languages; some references which can be accessed directly; hyperlinks between different parts of the paper. Finally, the articles in this special issue speak for themselves about probability education, and show that the community regains interest in probability education as a topic of its own interest. Future generations of students might thank the authors as they would also profit by understanding statistics better.

4. Perspectives for the Future

We end with some assertions requiring further research endeavour in probability education:

- People use their experience in order to judge probabilities incompletely and – even worse – in a haphazard manner.
- People have difficulties in judging very small and very high probabilities especially if these are connected to adverse consequences.
- People are inclined to attribute equal chances to the – given or seen – possibilities, especially if there are just two.
- People attribute probabilities and process these into new ones neglecting even the most basic rules (e. g. all probabilities sum to 1).

We believe that sharing and testing ideas across different countries will help promote deeper understanding. In particular, further empirical testing using shared instruments will yield deeper insights.

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LINKS

Working Group 5 on Stochastic Thinking at CERME 7: http://www.cerme7.univ.rzeszow.pl/?id=call-for-papers.

Topic Study Group 13 on "Probability" at ICME 11: tsg.icme11.org/tsg/show/14; see also the website of IASE at: www.stat.auckland.ac.nz/~iase/publications.php.

Topic Study Group 14 on "Statistics" at ICME 11: tsg.icme11.org/tsg/show/15.

Joint ICME/IASE study: www.ugr.es/~icmi/iase_study/.

SOFTWARE

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ABSTRACT

In the topic study group on probability at ICME 11 in 2008 a wide variety of ideas on probability education were presented, representing research across the world, including English, Spanish, French and German-speaking countries. In an international, collaborative, electronically supported project after the conference, all the key papers have been developed much further, innovatively incorporating electronic techniques. These techniques have been available for much of this century but hardly used in conveying research information. Following McLuhan, the medium of research has influenced the message (massage) of the results and - not surprisingly - the research has changed its character during this process. This presentation therefore presents a summary of the main threads of research in probability education across the multi-lingual world. It also comments on the influence of electronic communication in supporting such an exciting venture and its future possibilities. The summary will help researchers to build ideas the future; we believe that this paper will also enable developments relevant for other areas of research too.

Keywords: Research, probability education, electronic publishing, interactivity.