Applications of time-series models to ruin theory with dependent classes of business

Wat, Kam Pui
Department of Statistics and Actuarial Science
The University of Hong Kong
Pokfulam Road
Hong Kong
E-mail: kpwat@hku.hk

Li, Wai Keung
Department of Statistics and Actuarial Science
The University of Hong Kong
Pokfulam Road
Hong Kong
E-mail: hrntlwk@hku.hk

Yuen, Kam Chuen
Department of Statistics and Actuarial Science
The University of Hong Kong
Pokfulam Road
Hong Kong
E-mail: kcyuen@hku.hk

Introduction

Significance of time series analysis comes from the possibility to explain data from the past information, as well as the capability to perform forecasting. The dynamic of an order one autoregressive (i.e. AR(1)) model, a traditionally direct and simple time series model, captures the dependence relationship between two successive quantities. This autoregressive class of time series gives insights into the modelling of time-dependent quantities such as the aggregate claim amounts of an insurance company in a series of consecutive time periods.

In practice, the total claim amount of an insurance company in a certain time period may depends on the total claim amount in the previous time period to a certain extent. This phenomenon may be due to the fact that part of the actual claim payment may be delayed to the next time period for some reason. Gerber (1982) investigated the AR(1) model for claim amounts in the context of ruin theory which is a classical branch in actuarial science. Although there were quite a number of research papers applying time series models to some actuarial problems since then, further actuarial research in this direction remains to be studied.

The model

In recent years, researchers tried to extend the AR(1) model in various ways. Yang and Zhang (2003) included a constant interest risk in the model. Wan et al. (2005) and Zhang et al. (2007) extended the univariate time series to multivariate time series. In addition to the above-mentioned two extensions, more general time series risk models with dependent classes of insurance business are proposed in this paper.

Here, we adopt the following assumptions of Wan et al. (2005):

- Policies remain in force for an unlimited length of time.
- There are m classes of business in total.
- Each class of business has its own premium rates and claim amounts.
- Premiums are paid at the start of each period, and the amount is supposed to be a constant throughout the life of the policy.
- The total premiums paid in each period in class j is c_j, where $j = 1, 2, \cdots, m$.
- Y_{ji} is the total amount of claims incurred by the class j policies in the period i.
- The events causing Y_{ji} will cause further claims in the future periods, and not only in class j but also in other classes. This is the dependence structure of the model.
- W_{ji} is the total amount of claims paid on behalf of the class j policies in period i. It consists of Y_{ji} and a linear combination of all the previous claims in all classes, that is, a linear combination of all Y_{hk}'s for $h = 1, 2, \cdots, m$ and $k = 1, 2, \cdots, i-1$.
- $\mathbf{Y}_i = (Y_{1i}, Y_{2i}, \cdots, Y_{mi})'$. It is assumed that $\{\mathbf{Y}_1, \mathbf{Y}_2, \cdots\}$ is a sequence of independent and identically distributed (i.i.d.) non-negative random vectors having finite means and covariance matrices.
- $\mathbf{W}_i = (W_{1i}, W_{2i}, \cdots, W_{mi})'$. It is assumed that $\{\mathbf{W}_1, \mathbf{W}_2, \cdots\}$ is a sequence of dependent vectors. The dependence of these vectors are to be modelled by time series.

Within this set-up, a fixed interest rate r per period is also introduced to the model. Such an inclusion reflects the fact that insurance company can at least earn some interest from its assets through investing in some fixed income securities. As usual, it is assumed that premiums are received at the start of every period and claims are paid at the end of every period. Hence, the surplus process for class j can be formulated as

$$\begin{aligned} U_{jn} &= u_j(1+r)^n + c_j \cdot s_{\overline{n}|r} - S_{jn} \\ &= u_j(1+r)^n + c_j \cdot s_{\overline{n}|r} - \sum_{i=1}^{n}(1+r)^{n-i} W_{ji}, \end{aligned}$$

for $n = 1, 2, \cdots$, where U_{jn} denotes the surplus of class j at the end of period n with $U_{j0} = u_j$; S_{jn} is the total amount of claims in class j in period n; u_j is the initial surplus of class j; and $s_{\overline{n}|r} = ((1+r)^n - 1)/r$ is a standard actuarial notation for the accumulated annuity factor. As a result, the aggregate surplus process $\{U_n\}_{n=0,1,2,\cdots}$ at the end of period n, is

$$\begin{aligned} U_n &= \sum_{j=1}^{m} U_{jn} \\ &= u(1+r)^n + c \cdot s_{\overline{n}|r} - S_n, \end{aligned}$$

for $n = 1, 2, \cdots$. Note that

$$u = \sum_{j=1}^{m} u_j, \quad c = \sum_{j=1}^{m} c_j,$$

$$S_n = \sum_{j=1}^{m} S_{jn} = \sum_{j=1}^{m}\sum_{i=1}^{n}(1+r)^{n-i}W_{ji} = \sum_{i=1}^{n}(1+r)^{n-i}\sum_{j=1}^{m}W_{ji} = \sum_{i=1}^{n}(1+r)^{n-i}\mathbf{1}'\mathbf{W}_i,$$

where $\mathbf{1}$ is a column vector of 1's.

The relationship between two consecutive surpluses can be written as

$$\begin{aligned} U_n &= U_{n-1}(1+r) + c(1+r) - \sum_{j=1}^{m} W_{jn} \\ &= U_{n-1}(1+r) + c(1+r) - \mathbf{1}'\mathbf{W}_n, \end{aligned}$$

for $n = 1, 2, \cdots$. Denote $v = (1+r)^{-1}$ be the discount factor for one period. Then, the above relationship can be rewritten as

$$U_n = v^{-1}U_{n-1} + v^{-1}c - \mathbf{1}'\mathbf{W}_n, \tag{1}$$

for $n = 1, 2, \cdots$.

In the rest of the paper, we use vector time series models or multiple time series models to describe the vector claim amount $\mathbf{W}_n$ with innovation $\mathbf{Y}_n$ in period n. A modified surplus process $\{\hat{U}_n\}_{n=0,1,2,\cdots}$ is formed by a slight adjustment to $\{U_n\}_{n=0,1,2,\cdots}$, so that the relationship between the two consecutive modified surpluses is

$$\hat{U}_n = v^{-1}\hat{U}_{n-1} + v^{-1}c - \varepsilon_n, \tag{2}$$

for $n = 1, 2, \cdots$, where the univariate random variable ε_n varies from model to model depending on the time series parameters, but it is just a new innovation. Note that ε_n's are independent and identically distributed (i.i.d.) as well.

Vector autoregressive models

Wan et al. (2005) and Zhang et al. (2007) formulated the vector $\mathbf{W}_n$ as VAR(1), a vector autoregressive model of order one. That is,

$$\mathbf{W}_n = \mathbf{A}\mathbf{W}_{n-1} + \mathbf{Y}_n,$$

where $\mathbf{A}$ is the autoregressive parameter matrix. Within this framework, it was shown that equation (1) can be transformed to equation (2) in the following way. Let $\mathbf{p}$ be a column vector which fits the dimension of the equation. This vector is crucial to develop a link between the original surplus and the modified surplus.

From equation (1), we have

$$\begin{aligned} U_n &= v^{-1}U_{n-1} + v^{-1}c - \mathbf{1}'\mathbf{W}_n, \\ U_n + \mathbf{p}'\mathbf{W}_n &= v^{-1}U_{n-1} + v^{-1}c - (\mathbf{1}-\mathbf{p})'(\mathbf{A}\mathbf{W}_{n-1} + \mathbf{Y}_n), \\ U_n + \mathbf{p}'\mathbf{W}_n &= v^{-1}\left[U_{n-1} - v(\mathbf{1}-\mathbf{p})'\mathbf{A}\mathbf{W}_{n-1}\right] + v^{-1}c - (\mathbf{1}-\mathbf{p})'\mathbf{Y}_n. \end{aligned}$$

Setting $\mathbf{p}' = -v(\mathbf{1}-\mathbf{p})'\mathbf{A}$ would yield $\mathbf{p}' = -\mathbf{1}'(\mathbf{I}-v\mathbf{A})^{-1}v\mathbf{A}$ and $(\mathbf{1}-\mathbf{p})' = \mathbf{1}'(\mathbf{I}-v\mathbf{A})^{-1}$. Therefore, equation (2) becomes

$$\hat{U}_n = v^{-1}\hat{U}_{n-1} + v^{-1}c - \mathbf{1}'(\mathbf{I}-v\mathbf{A})^{-1}\mathbf{Y}_n,$$

where the modified surplus is

$$\hat{U}_n = U_n - \mathbf{1}'(\mathbf{I}-v\mathbf{A})^{-1}v\mathbf{A}\mathbf{W}_n,$$

and

$$\varepsilon_n = \mathbf{1}'(\mathbf{I}-v\mathbf{A})^{-1}\mathbf{Y}_n.$$

Note that this result is a special case of Zhang et al. (2007).

We now generalize the result to $\mathbf{W}_n \sim \text{VAR}(2)$, that is,

$$\mathbf{W}_n = \mathbf{A}_1\mathbf{W}_{n-1} + \mathbf{A}_2\mathbf{W}_{n-2} + \mathbf{Y}_n,$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the autoregressive parameter matrices. In this case, parallel to the previous derivation, one can show that equation (1) can be transformed to equation (2) as

$$\hat{U}_n = v^{-1}\hat{U}_{n-1} + v^{-1}c - \mathbf{1}'\left(\mathbf{I} - v\mathbf{A}_1 - v^2\mathbf{A}_2\right)^{-1}\mathbf{Y}_n,$$

where the modified surplus is

$$\hat{U}_n = U_n - \mathbf{1}'\left(\mathbf{I} - v\mathbf{A}_1 - v^2\mathbf{A}_2\right)^{-1}\left(v\mathbf{A}_1 + v^2\mathbf{A}_2\right)\mathbf{W}_n - \mathbf{1}'\left(\mathbf{I} - v\mathbf{A}_1 - v^2\mathbf{A}_2\right)^{-1} v\mathbf{A}_2\mathbf{W}_{n-1},$$

and

$$\varepsilon_n = \mathbf{1}'\left(\mathbf{I} - v\mathbf{A}_1 - v^2\mathbf{A}_2\right)^{-1}\mathbf{Y}_n.$$

In fact, by adopting this approach, the time series risk model can be further generalized to the case of order p, that is, the VAR(p) model. Note that, for the results obtained above, it is also assumed that parameter matrices are chosen such that the vector time series model is both stationary and invertible.

Vector moving average models

Besides the vector autoregressive models, it is also natural to consider the vector moving average models to describe the vector $\mathbf{W}_n$.

First, consider the simplest case of order one with $\mathbf{W}_n \sim \text{VMA}(1)$, that is,

$$\mathbf{W}_n = \mathbf{Y}_n - \mathbf{M}\mathbf{Y}_{n-1},$$

where $\mathbf{M}$ is the moving average parameter matrix. Then, we use the idea of transforming equation (1) to equation (2) to obtain similar results. Let $\mathbf{q}$ be a column vector which fits the dimension of the equation. It follows from equation (1) that

$$\begin{aligned}
U_n &= v^{-1}U_{n-1} + v^{-1}c - \mathbf{1}'\mathbf{W}_n, \\
U_n &= v^{-1}U_{n-1} + v^{-1}c - \mathbf{1}'\mathbf{Y}_n + \mathbf{1}'\mathbf{M}\mathbf{Y}_{n-1}, \\
U_n + \mathbf{q}'\mathbf{Y}_n &= v^{-1}\left(U_{n-1} + v\mathbf{1}'\mathbf{M}\mathbf{Y}_{n-1}\right) + v^{-1}c - (\mathbf{1} - \mathbf{q})'\,\mathbf{Y}_n.
\end{aligned}$$

Put $\mathbf{q}' = \mathbf{1}'v\mathbf{M}$. Then, equation (2) becomes

$$\hat{U}_n = v^{-1}\hat{U}_{n-1} + v^{-1}c - \mathbf{1}'\left(\mathbf{I} - v\mathbf{M}\right)\mathbf{Y}_n,$$

where the modified surplus is

$$\hat{U}_n = U_n + \mathbf{1}'v\mathbf{M}\mathbf{Y}_n,$$

and

$$\varepsilon_n = \mathbf{1}'\left(\mathbf{I} - v\mathbf{M}\right)\mathbf{Y}_n.$$

For $\mathbf{W}_n \sim \text{VMA}(2)$, we define

$$\mathbf{W}_n = \mathbf{Y}_n - \mathbf{M}_1\mathbf{Y}_{n-1} - \mathbf{M}_2\mathbf{Y}_{n-2},$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the moving average parameter matrices. In this case, by applying similar steps, equation (1) can be transformed to equation (2) as

$$\hat{U}_n = v^{-1}\hat{U}_{n-1} + v^{-1}c - \mathbf{1}'\left(\mathbf{I} - v\mathbf{M}_1 - v^2\mathbf{M}_2\right)\mathbf{Y}_n,$$

where the modified surplus is

$$\hat{U}_n = U_n + \mathbf{1}'\left(v\mathbf{M}_1 + v^2\mathbf{M}_2\right)\mathbf{Y}_n + \mathbf{1}'v\mathbf{M}_2\mathbf{Y}_{n-1},$$

and

$$\varepsilon_n = \mathbf{1}'\left(\mathbf{I} - v\mathbf{M}_1 - v^2\mathbf{M}_2\right)\mathbf{Y}_n.$$

Similarly, the risk model can also be generalized to the case of order q, that is, the VMA(q) model. Again, the above results requires the assumption that parameter matrices are chosen such that the vector time series model is both stationary and invertible.

Risk theory results

An important quantity in risk theory is the so-called adjustment coefficient R which is defined to be the smallest positive root of the equation

$$\mathrm{E}\left[e^{-R(v^{-1}c-\varepsilon_1)}\right] = 1,$$

or equivalently, $e^{-Rv^{-1}c}\cdot\mathrm{E}\left(e^{R\varepsilon_1}\right) = 1$, or $\ln M_{\varepsilon_1}(R) = Rv^{-1}c$ where $M_X(\cdot)$ is the moment generating function (m.g.f.) of the random variable X, that is, $M_X(t) = \mathrm{E}\left(e^{tX}\right)$. Furthermore, the net profit condition requires

$$v^{-1}c = (1+\eta)\mathbf{1}'\mathrm{E}(\mathbf{W}_1),$$

with the security loading $\eta > 0$. This condition says that the total premiums collected from all classes should be greater than the expected total claim amounts for all classes. If this assumption is not satisfied, then ruin is certain. Note that the condition implies that

$$v^{-1}c > \mathrm{E}(\varepsilon_1).$$

Define the time of ruin as $T = \min\{n : U_n < 0\}$. Note that $T = \infty$ if the set is empty. Then, the ruin probability with initial claim w is defined to be

$$\psi(u,w) = \Pr(T < \infty).$$

Theorem 1. *Given the modified initial surplus $\hat{u} = \hat{U}_0$, an upper bound of the ruin probability is given by*

$$\psi(u,w) \le \frac{e^{-R\hat{u}}}{\mathrm{E}\left(e^{-Rv^T\hat{U}_T}\,\middle|\, T<\infty\right)}. \tag{3}$$

Note that the equality in (3) holds if $v = 1$, that is, when $r = 0$.

Proof. Following the method of Zhang et al. (2007), one can show that $\{e^{-Rv^n\hat{U}_n}\}_{n=0,1,2,\cdots}$ is an $\mathcal{F}_n$-supermartingale, and hence completes the proof using similar arguments. □

Corollary 2. *As a result of Theorem 1, a larger but computable upper bound is given by*

$$\psi(u,w) \le e^{-R\hat{u}}.$$

Proof. At the time of ruin T, $U_T < 0$ and $\hat{U}_T < 0$ by regularity conditions. Therefore, the denominator $\mathrm{E}\left(e^{-Rv^T\hat{U}_T} \,\middle|\, T < \infty\right)$ is greater than one. It completes the proof. □

Simulation studies

To investigate the difference between the VAR(1) model and VMA(1) model, simulation studies are carried out for the finite time ruin probability of an insurance surplus process with $m = 2$ classes of business. In order to have a fair comparison, the paramter matrices are chosen in the way such that both models have the same mean.

Recall that for the VAR(1) model,

$$\mathbf{W}_n = \mathbf{A}\mathbf{W}_{n-1} + \mathbf{Y}_n.$$

By the stationarity condition, the mean is

$$\mathrm{E}(\mathbf{W}_n) = (\mathbf{I} - \mathbf{A})^{-1}\mathrm{E}(\mathbf{Y}_n).$$

On the other hand, the VMA(1) model is

$$\mathbf{W}_n = \mathbf{Y}_n - \mathbf{M}\mathbf{Y}_{n-1},$$

with mean

$$\mathrm{E}(\mathbf{W}_n) = (\mathbf{I} - \mathbf{M})\mathrm{E}(\mathbf{Y}_n).$$

Then, for a specific distribution of $\mathbf{Y}_n$ with finite mean, it is required to set

$$\mathbf{I} - \mathbf{M} = (\mathbf{I} - \mathbf{A})^{-1}.$$

Suppose that the moving average parameter matrix is chosen to be

$$\mathbf{M} = \begin{bmatrix} -0.8 & -0.1 \\ -0.1 & -0.8 \end{bmatrix}.$$

Then,

$$\mathbf{I} - \mathbf{M} \;=\; \begin{bmatrix} 1.8 & 0.1 \\ 0.1 & 1.8 \end{bmatrix},$$

so the autoregressive parameter matrix should be

$$\begin{aligned} \mathbf{A} &= \mathbf{I} - (\mathbf{I} - \mathbf{M})^{-1} \\ &= \begin{bmatrix} 0.4427 & 0.0310 \\ 0.0310 & 0.4427 \end{bmatrix}. \end{aligned}$$

For the innovation term $\mathbf{Y}_n$, it is necessary to use a distribution with non-negative support with finite mean and variance. One possible choice is the absolutely continuous bivariate exponential distribution (ACBVE) (see Block and Basu (1974)). In the simulation study, we set $\mathbf{Y}_n \sim \mathrm{ACBVE}(0.3, 0.3, 1)$.

With the above two time series models, simulations are performed with initial surplus $u \in \{0, 10, 20, \cdots, 100\}$, premium $c \in \{1.0, 1.5, 2.0, \cdots, 5.0\}$ and interest rate $r \in \{0.00, 0.01, 0.02, \cdots, 0.07\}$. The number of periods in consideration for the finite time ruin probability is $1,000$.

Ruin probability for VAR(1) ***model (*** r ***by*** u ***at*** $c = 2.5$***)***

	20		30		40		50	
0	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)
0.01	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)
0.02	0.9995	(0.0022)	0.9944	(0.0072)	0.9481	(0.0225)	0.7826	(0.0446)
0.03	0.9665	(0.0192)	0.7834	(0.0388)	0.4449	(0.0584)	0.1573	(0.0386)
0.04	0.7913	(0.0359)	0.3875	(0.0516)	0.1076	(0.0292)	0.0162	(0.0129)
0.05	0.5261	(0.0530)	0.1414	(0.0321)	0.0183	(0.0125)	0.0018	(0.0044)
0.06	0.3134	(0.0474)	0.0468	(0.0208)	0.0043	(0.0073)	0.0001	(0.0010)
0.07	0.1750	(0.0400)	0.0158	(0.0124)	0.0004	(0.0020)	0.0000	(0.0000)

Ruin probability for VAR(1) ***model (*** r ***by*** u ***at*** $c = 4.0$***)***

	20		30		40		50	
0	0.3258	(0.0458)	0.2192	(0.0423)	0.1429	(0.0323)	0.0887	(0.0281)
0.01	0.0725	(0.0263)	0.0204	(0.0148)	0.0059	(0.0071)	0.0009	(0.0032)
0.02	0.0285	(0.0155)	0.0042	(0.0057)	0.0003	(0.0017)	0.0000	(0.0000)
0.03	0.0115	(0.0101)	0.0020	(0.0045)	0.0001	(0.0010)	0.0000	(0.0000)
0.04	0.0039	(0.0058)	0.0004	(0.0020)	0.0000	(0.0000)	0.0000	(0.0000)
0.05	0.0017	(0.0045)	0.0002	(0.0014)	0.0000	(0.0000)	0.0000	(0.0000)
0.06	0.0018	(0.0039)	0.0000	(0.0000)	0.0000	(0.0000)	0.0000	(0.0000)
0.07	0.0007	(0.0026)	0.0000	(0.0000)	0.0000	(0.0000)	0.0000	(0.0000)

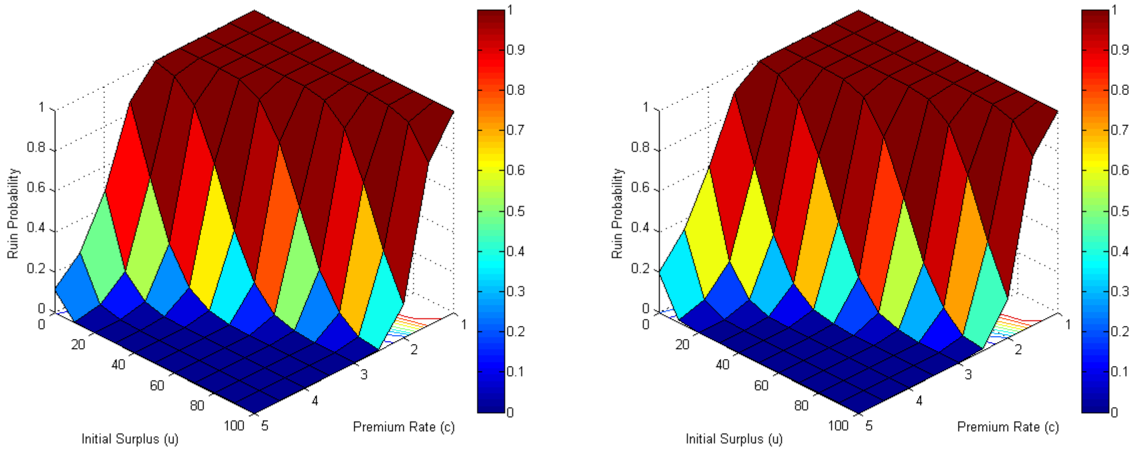
Ruin probability for VMA(1) ***model (*** r ***by*** u ***at*** $c = 2.5$***)***

	20		30		40		50	
0	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)
0.01	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)	1.0000	(0.0000)
0.02	0.9998	(0.0014)	0.9959	(0.0060)	0.9592	(0.0200)	0.8150	(0.0451)
0.03	0.9754	(0.0160)	0.8194	(0.0348)	0.4951	(0.0573)	0.1907	(0.0410)
0.04	0.8314	(0.0385)	0.4481	(0.0540)	0.1394	(0.0319)	0.0233	(0.0150)
0.05	0.5932	(0.0527)	0.1876	(0.0390)	0.0275	(0.0149)	0.0029	(0.0054)
0.06	0.3774	(0.0482)	0.0678	(0.0250)	0.0066	(0.0087)	0.0001	(0.0010)
0.07	0.2316	(0.0427)	0.0241	(0.0149)	0.0014	(0.0038)	0.0000	(0.0000)

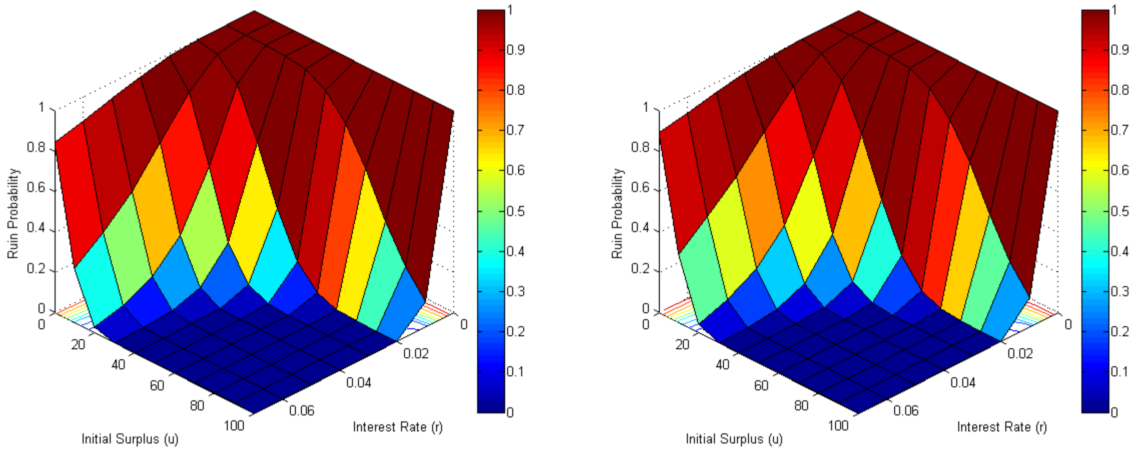
Ruin probability for VMA(1) ***model (*** r ***by*** u ***at*** $c = 4.0$***)***

	20		30		40		50	
0	0.3625	(0.0461)	0.2469	(0.0430)	0.1598	(0.0373)	0.0990	(0.0307)
0.01	0.0975	(0.0305)	0.0276	(0.0164)	0.0078	(0.0079)	0.0017	(0.0043)
0.02	0.0409	(0.0194)	0.0068	(0.0069)	0.0007	(0.0029)	0.0000	(0.0000)
0.03	0.0202	(0.0129)	0.0027	(0.0053)	0.0002	(0.0014)	0.0000	(0.0000)
0.04	0.0075	(0.0081)	0.0006	(0.0024)	0.0002	(0.0014)	0.0000	(0.0000)
0.05	0.0044	(0.0067)	0.0004	(0.0020)	0.0000	(0.0000)	0.0000	(0.0000)
0.06	0.0027	(0.0049)	0.0000	(0.0000)	0.0000	(0.0000)	0.0000	(0.0000)
0.07	0.0020	(0.0043)	0.0001	(0.0010)	0.0000	(0.0000)	0.0000	(0.0000)

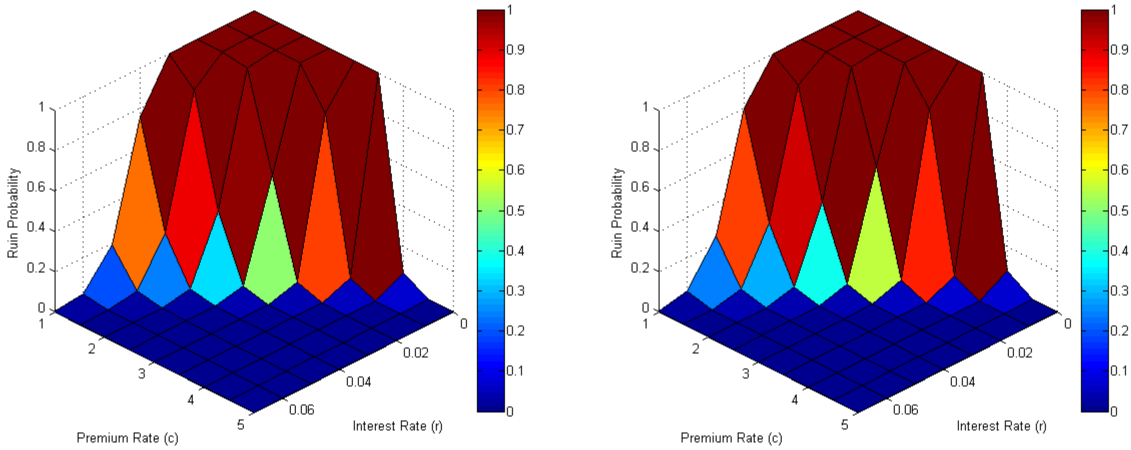
Model comparison for ruin probability at $r = 0.02$ ***(Left:*** VAR(1)***, Right:*** VMA(1)***)***



Model comparison for ruin probability at $c = 3.0$ ***(Left:*** VAR(1)***, Right:*** VMA(1)***)***



Model comparison for ruin probability at $u = 60$ ***(Left:*** VAR(1)***, Right:*** VMA(1)***)***



As expected, in both models, the ruin probability decreases as the initial surplus increases, or as the premium amount increases, or as the interest rate increases. On the other hand, the two time series models tend to yield similar results. One of the reasons may be due the restriction of setting the same mean. However, it is worthwhile to consider the difference in model interpretations. Autoregressive models can refer to the cases of delayed claims, while moving average models capture the trend of the noise terms, which are regarded as the newly incurred claims in every period. Hence, different models can suit several practical needs.

Summary

In this paper, we extend the traditional time-series insurance risk models in several ways. The inclusion of interest rates captures the concept of time value of money in finance. The extension of univariate time series to multiple time series allows the modelling of the time dependence structure across different but related business classes. Specifically, we present a general method to extend the existing VAR(1) risk model to the VAR(p) risk model, and propose the vector moving average risk models to handle the dependence between business classes. Most importantly, with all the aforementioned extensions, an exponential bound can still be obtained for the ruin probability of the risk process.

For further research, it is also tempting to investigate the generalized vector autoregressive moving average (VARMA(p, q)) models, which is a possible extension of the above analysis. It is believed that by using similar arguments, a similar result can be worked out. Another possible research problem is to consider certain dependence structure among claim numbers of all classes. In this case, vector integer time series analysis may be a possible tool for modelling the relation between claim numbers.

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RÉSUMÉ

K. P. Wat is a Ph.D. candidate at the Department of Statistics and Actuarial Science, The University of Hong Kong, under the joint supervision of Professor W. K. Li and Professor K. C. Yuen.